Klasa .... Rok ....

Oddział .... Półrocze ....

28

$$\int_{-\infty}^{+\infty} e^{-\frac{\nu\delta^2}{2}} d\delta = \sqrt{\frac{2\pi}{\nu}} \qquad \delta\sqrt{\frac{\nu}{2}} = x$$

$$\frac{\int_0^{\infty} \delta e^{-\nu\frac{\delta^2}{2}} d\delta}{\int_0^{\infty} e^{-\nu\frac{\delta^2}{2}} d\delta} = \frac{\int_0^{\infty} x e^{-x^2} dx}{\sqrt{\frac{\nu}{2}}\int_0^{\infty} e^{-x^2} dx} = \frac{\frac{e^{-x^2}}{2}\Big|^0}{\sqrt{\frac{\nu}{2}}\cdot\frac{\pi}{2}} = \sqrt{\frac{2}{\nu\pi}}$$

$$h = \cancel{\frac{32}{3} \frac{\lambda^{+3}}{\pi^3} \cdot \varepsilon^2} \quad \frac{32}{3}\frac{\pi^3}{\lambda}\varepsilon^2$$

$$\varepsilon = 7.2 \cdot 10^{-3}$$

$$\frac{32 \cdot \cancel{31} \cdot 500 \,|\, 10^{-6}}{\cancel{3} \cdot 0.00006} = \frac{16 \cdot 10^{-3}}{0.00016}$$

$$= 2.7 \cdot 10^{2}$$

$$\frac{32}{3} \frac{\cancel{30} \cdot 10}{0.00006} \left[0.00029 \cdot 2.7 \cdot 10^{-4}\right]^2$$

$$8 \cdot 10^{-8}$$

$$= \frac{3.2 \cdot 10^2 \cdot 64 \cdot 10^{-16}}{6 \cdot 10^{-5}} = \underline{\underline{3 \cdot 10^{-8}}}$$

$$\frac{0.03 \cdot 0.434}{0.013}$$

$$10\ km = 10.000\,00$$

$$e^{-3 \cdot 10^{-2}}$$

$$e^{-hx} = \frac{1}{2}$$

$$h x \,{}^{10}\lg e = \lg 2$$

$$x = \frac{\lg 2}{h \,{}^{10}\lg e} = \frac{0.301}{2.7 \cdot \cancel{28}\, 0.434}$$

$$\begin{array}{r} 6375 \\ 4314 \\ \hline 0689 \end{array} \qquad \begin{array}{r} 479 \\ -069 \\ \hline 410 \end{array} \Big| 026$$

$p + a\rho^2 = RT\rho\,\varphi\left(\frac{\rho}{\rho_0}\right)$

$1 + 2a\rho\frac{\partial\rho}{\partial p} = RT\left[\varphi + \frac{\rho}{\rho_0}\varphi'\right]\frac{\partial\rho}{\partial p} \quad \Big|\ \frac{\partial p}{\partial T}$

$2a\rho\frac{\partial\rho}{\partial T} = R\rho\varphi + RT\left[\varphi + \frac{\rho}{\rho_0}\varphi'\right]\frac{\partial\rho}{\partial T} \quad -\frac{\partial\rho}{\partial p}$

$\frac{\partial\rho}{\partial T} = -R\rho\varphi\,\frac{\partial\rho}{\partial p}$

$-\varphi\left(\frac{\rho}{\rho_0}\right) = \frac{1}{R\rho}\left(\frac{\frac{\partial\rho}{\partial T}}{\frac{\partial\rho}{\partial p}}\right)$

$a = -\left(\frac{T\frac{\partial\rho}{\partial T}}{\rho^2\frac{\partial\rho}{\partial p}}\right)_{p=0}$

0 014646 · · ·6186 · 4825 = 0 025657

14646 · 1239 · 536 = 16421

$v_{100} = v_0(1{\cdot}1465 + \cdot 309 + \cdot 161) = 1{\cdot}1935$

Pentan p 59

0°	$\frac{1}{\rho}\frac{\partial\rho}{\partial p} =$	229 10^{-6}	146
20°		318	164
40°		416	
60°		486	
80		610	
100		714	00257

$\frac{a_0}{a_{100}} = \frac{273}{373}\ \frac{257}{146}\ \frac{229}{714}\ \frac{1}{(1{\cdot}1935)^2}$

$\frac{273.146}{229} \cdot \frac{714}{257.373}$

4362 1644 8537 4543 | 3598 4099 5717 1536 4950

4362 4099 3598 2059 7434 4625

5717 1644 8537 1930 7434

4543 4950 9593

0·9106

165701 174 1831

~~0·0014646~~ 0·001028 256 0001384

1·1028 178 1·1206

Isohd $\beta = \cdot 0_4 770\,(1 + 0{\cdot}0_2 65701 + 0{\cdot}0_4 174\, t^2)$

$\frac{a_0}{a_{100}} = \frac{1{\cdot}831}{\cancel{1{\cdot}8}} \cdot \frac{273}{373} \cdot \frac{1028}{1384} \cdot \frac{1}{(1{\cdot}1206)^2}$

4362 0120 4482 2627 7109

5717 1412 0988 8117

0494 0988

7109 8117 8992

0·793

$$\cancel{p} + a\rho^2 = RT\, \varphi\left(\frac{v}{b}\right)$$

$$= RT\, \varphi\left(\frac{\rho}{\rho_0}\right)$$

$$a\rho^2 = RT\, \varphi\left(\frac{\rho}{\rho_0}\right)$$

$$\frac{\partial\rho}{\partial T} \quad \frac{\partial\rho}{\partial p} \quad \Big| \quad 1 + 2a\rho \frac{\partial\rho}{\partial p} = RT\, \varphi'\left(\frac{\rho}{\rho_0}\right) \frac{1}{\rho_0} \frac{\partial\rho}{\partial p}$$

$$2a\rho \frac{\partial\rho}{\partial T} = R\varphi\left(\frac{\rho}{\rho_0}\right) + \frac{RT}{\rho_0} \varphi'\left(\frac{\rho}{\rho_0}\right) \frac{\partial\rho}{\partial T}$$

$$\frac{\partial\rho}{\partial T} = -R\varphi \cdot \frac{\partial\rho}{\partial p}$$

$$-\varphi\left(\frac{\rho}{\rho_0}\right) = \frac{1}{R} \frac{\frac{\partial\rho}{\partial T}}{\frac{\partial\rho}{\partial p}}$$

$$a = -\left(\frac{T \frac{\partial\rho}{\partial T}}{\rho^2 \frac{\partial\rho}{\partial p}}\right)_{p=0}$$

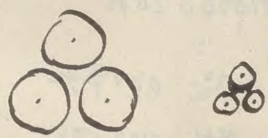
$$\mu_v : \mu_{t_0, v} = \sqrt{t} : \sqrt{t_0}$$

$$\mu_{t_0, v} = \psi\left(\frac{\rho}{\rho_0}\right)$$

$u \qquad \kappa u$

$\dfrac{u^2}{\frac{\partial u}{\partial z}} \qquad \dfrac{\kappa^2 u^2}{\kappa \frac{\partial u}{\partial z}} \qquad \sim \kappa$

Gleiche Raumerfüllung & gl. Dimens.



n

μ

$$F' = F\frac{n}{n} \cdot n^2$$

$c \qquad \kappa c$

$u \qquad \kappa u$

$\kappa^2 \quad \dfrac{\kappa^2 u^2}{\kappa^2 \quad \kappa \frac{\partial u}{\partial z}} \qquad \sim \kappa$

$$mc^2 = MC^2$$

$$\mu = \frac{mc}{\delta^2}\, \psi\left(\frac{\rho}{\rho_0}\right) = \sqrt{mT}\, \psi\left(\frac{v}{b}\right) = (mT)^{\frac{1}{2}}\, \psi\left(\frac{v}{b}\right)$$

$$\lg \mu = \tfrac{1}{2} \lg T + \lg \psi$$

$$\frac{1}{\mu}\frac{d\mu}{dT} = \frac{1}{2}\frac{1}{T} + \frac{1}{b}\frac{\psi'\left(\frac{v}{b}\right)}{\psi} \frac{dv}{dT} = \frac{1}{2}\frac{1}{T} + \boxed{\frac{\psi'\left(\frac{v}{b}\right)}{\psi\left(\frac{v}{b}\right)} \cdot \frac{v}{b}} \left(\frac{1}{v}\frac{dv}{dT}\right)$$

$\chi\left(\frac{v}{b}\right) \qquad \alpha$

$$\varphi\left(\frac{v}{b}\right) = \frac{\mu}{\sqrt{mT}}$$

$m = 200$

Hg	−21·0 (273)	0·01868	251·0	
	0°	0·01688	273	
	16·7	0·01575	289·7	
	99°	0·01227	372	
	196·7	0·01018		
	340·1	0·008975		

$\lg = 06856,\ 03822,\ 00795$

124°: 0·01171
154 : 0·01092
196·7 : 0·01018

2·46495
2·30103
4·76298
2·38149

19756
38149
81607

5705
9010
4·8715
2·43575

0889
43575
65315

Methylpropylketon	0°	0·00644
m = 86	100°	0·00238
Heptan	0°	00519
100	90°	00214
Oktan	0°	0·00703
114	120°	0·002075
C_2H_5OH	0°	0·01770
46	70°	0·005045
Amylalk. (inakt)	0°	0·08532
88	130°	0·00386
Butylalk	0°	0·05185
74	110	0·004545
Propylacetat	0°	0·00770
	100°	0·00250

$$\frac{1}{\mu}\frac{d\mu}{dT} = \frac{d}{dT}(\lg \mu) = \cancel{\tfrac{d\mu}{dt}}\ {}^{e}\lg_{10} \frac{d}{dT}({}^{10}\lg \mu)$$

−21·0°	2690	
0°	~~1976~~	2274
16·70	1976	
99°	0889	
124°	~~07078~~	0686
154°	0382	
196·70	0 0775	

~~3132~~
41·6 : 21·4 = 19·4
208 : 16·7 = 12·7
109 : 82·3 = 132·5
203 : 25 = 80
30 : 30 = 100
305 : 42·5 = 72·0

$$\frac{1}{\mu}\frac{d\mu}{dT} = 0·00110 \cdot 2·3026$$

23026
0·002533|29

~~46 : 99 = 465~~ 66 : 200 = 33

$$\frac{0033}{123} =$$

Benzol 0° 0·009025

78. 80° 0·00327

$V_t = V_0(1 + 0·0_2 11 7626\, t + 0·0_5 127755\, t^2 + 0·0_7 080648)$ 3

$(1 + 0·0_2 116\, t + 0·0_5 2226\, t^2)$

Petroleum 0° 0·02284

88 160° 0·00314

1·8921
2·4362
4·3283
2·16415

9554
~~1641~~
− 7913
0°

10° 1·8921
24518
43439
2·17195

8802
1720
7082
10°

20° 1·8921
4669
3590
2·1795

8122
1795
6327

30° 1·8921
4814
3735
2·18675

7497
1867
5630

7913
7082
6327
5630

83
75·5
70

−79
73

74·5

$\frac{20·5}{74·5} =$ 2118 / 1722 / 4396 = 275

99°
Hg

22·75°
C6H6

0°	902·5	153·5
10°	759	110
20°	649	87
30°	562	70
40°	492	55
50°	437	

$\frac{1}{r}\frac{dr}{dt} = 0·00253$

0·0150

9554 75 7
8802 68 ... 65·5
8122 63 6
7497 57 5
6920 52
6405

$65 \cdot 275 = \frac{11}{4}$

$715 : 4 = 18$

794

0·0065 . 2·3
130
195
0·01495

1·3590
0645
1·2945

7140
3476
0664

115·2

7610
1·3570
3476
·0056

103

$\alpha = 0·000\,18215$

$\alpha = 0·00116$
10
0·00126

Hg 99°

5705
4295

0·00269

$\frac{1}{\mu}\frac{d\mu}{dt}$ = − 0·00253

$-\frac{1}{2}\frac{1}{T}$ 0·00134

~~0·00119~~

− 0·00387 5877

α = 0·000182 2601

3276

213

CCl₄ 2273

273

29573

4709
5291

0·0033815

− 0·0150
0·0017

2

− 0·0167

0·00126

2227
1004
1223

13·2

Butylalk. ~~464~~ 213

74

10° 3872 5879
20° 2947 4694
30° 0·02266 3553 1753 1800
40° 1780 — — 2504 − 1823 0681
50 1409 1489 —
60 1136·5 0556 1958 8598 9557
70 926·5 9668 2024 7646
80 762 8820 2085 6735 6532
80 2504 6335 1489 8017 2145 5872
100 182 5345 1892
0681 9597 7779 2204 5075

952
911
863 884 875
797

82·270

1·8692 8692
2·4814 4955
4·3506 3647
30° 5092
3784
5224
3916
5353
4045
5478
4170
5599
4291
5717
4409

$\frac{20}{88}$ = 2·27

0·0008375
4711
0013086
− 252
0·00012834

0·00875
0·00128

9420
3617
3037
− 1072
1965

1572

91.52 8304
3010 4771
4569 0940
6731 4015
4711 2520

$2{\cdot}0568$ ~~4470~~ $1{\cdot}6628$ 7028

4362 $-\ 24055$ $2{\cdot}5353$ $-\ 09905$

$4{\cdot}4931$ 60045 $4{\cdot}1981$ 60375

333 70°

C_2H_5OH

143

60° $1{\cdot}6628$ 7720 50° 6628 8435

$2{\cdot}5224$ $-\ 0926$ 5092 $-\ 0860$

$4{\cdot}1852$ 6794 1720 7575

70° 6037 $653{\cdot}2$ 76 $\frac{26{\cdot}2}{77} = \cdot 4183$ $=34$ $63{\cdot}40$

60° 6794 78 8865

50° 7575 532

$$\frac{d}{dT}\left(\frac{\mu}{\sqrt{mT}}\right) = \frac{1}{\mu}\frac{d\mu}{dT} - \frac{1}{2}\frac{1}{T} = 0{\cdot}0077\ \ 1{\cdot}23$$

161

161

$\cdot 001771$

$0{\cdot}00104139$ 8021 6042

9935 8941 2460

2125 3010 4771

0001353 9972 3273

9935 $212{\cdot}5$

2480

1313

1167 $13{\cdot}1$

Amylalk. (68)

inact.

~~50~~°	0 1849	2669	2268₃	1400	154
60	1443	1593	2334₅	9859	166
70	1147	0597	2399	8198	100₅
80	923·5	9654	2461₅	7193	92₁
90	0 07575	8794	2522	6272	87₇
100	627·5	7996	2581	5395	79₈
110	529	7235	2638₅	4597	75₅
120	4505	6597	2694₅	3842	72₅
130	386	5806	2749	3117	

963

90 6532 $\frac{26}{90} = 29$ 874°

5092	5224	5353	5478	5599	5717	5832	5944	6053
9445	9445	9445	9445	9445	9448	9445	9445	9445
4537	4669	4798	4923	5044	5162	5277	5389	5498

42. 22 22

963 921 899 877

50 29

00913 .23
1826
2739

0·0210

0·00092410
0460
307
0·001277

9400	8800
3010	4771
4221	1299
6631	4870
460	307

0·0210
0·00128

3222
1072
215

164

Dimethyläthylcarbinol

$C_5H_{12}O$ 16
12
60
88

50°	0·01457	1641	2268	9373	138
60°	10.775	0324	2334	7990	120 ·8 — 11°
70°	·830	9191	2399	6792	107 ·20
80°	657·5	8179	2461	5718	100
90°	530	7243	2522	4721	93
100°	434	6375	2581	3794	

5322

$\frac{26}{111} = 2·4$ 72·4°

70°	75°	80°	
113·5	107	103·5	100

110·2

·12

0·0112 . 2·3
224
336
0·02576

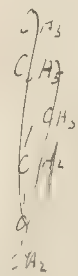
0·00106608
2554
222
0·001543

0·02576
0·00154

4109
·875
2234

16·7

8597	7194
3010	4771
2405	1498
4072	3463
2554	222



Cl_2 56·41 0·00706 8488
80 273 − 2104
329·4 6384

9031 46·19 0·00772 8876 445·6 : 447
5177 273 2036 10·2 148 = 1703
4208 319·2 6840 651·2 443 0435 3·35
5268

9031 35·86 0·00848 9284 478 460
5041 273 19653 104
4072 309 7318
9031 26 0·00934 9703 4·96
4900 299 1894 491
3931 ~~325~~ 7809 94
9031
4757
3788

56·41
− 3·36
53·05 0·0044

0·0010622 7243 4486
199 7010 4771
0·0012612 2735 4893
− 200
0·001235 2988 7150
199 200 366·23
232
1098
8·42

$\frac{0·0044}{0·0012}$ = 3·66 8·42

$CHCl_3$ 8451 ⟷ Butylalk.
1065 − 2569 74
3
119·5

0777
4362
5139

[illegible]
88

0°	0·1418	1517	1895₅	9622	961₃₅	264	·8
10°	0·0786	8954	1973₅	6981	697₂₅	230	30–216
20	464₂	0667	2049₅	4618	461	197	·3
30	300	4771	2122₅	2649	264	154	
40	204₄	3104	2193₅	0971	090₄	4	
50°	145 7	1635	2262	9373	936₇		

$\frac{63}{225} = \frac{7}{25} = 2·8$

9445 9445 9445 9445 9445 9445
~~4346 4502 4654 4800 4942 5079~~

~~3791 3947~~ 4099 4245 | 4387 | 4524 |

(17·2°)

4362 4518 4669 4814 4955 5092
9445 9445 9445 9445 9445 9445

225

3807 3963 4114 4259 4400 4537
1517 8954 6667 4771 3104 1635
1903₅ 1981₅ 2057 2129₅ 2200 2268₅

9613₅ | 6972₅ | 4610 | 2641₆ | 0904 | 9366₅

0·0225
0·00114

0·001066
~~667~~
12
0·00114

9010	4771
2355	4710
2465	1498
7830	0979
607	125

~~20~~ 7

46

Ontyldk

0°	0·05185	7147	1903	5244	135
10	3872	5879	1981	3898	126
20	2987	4694	2057	2037	121
30	2266	3552	2129	1423	

5628 148 · 141

$\frac{38}{140} = 2{\cdot}70$

0·0138 / 0·000845

1492 / 9132 / 1226

1399 / 9269 / 213

8692 / 4346

3038 / 7147 / 1519 / 5628

16·3

375

Amylalk

0°	0·08532	9310	1903	7407	160·6
	6000	7782	1981	5801	148·2
	4341	6376	2057	4319	138·9
	3206	5059	2129	2930	

12 — 154

$\frac{50}{152} = 3{\cdot}7$ 13·70

$\frac{0{\cdot}0150}{0{\cdot}00094} = 16{\cdot}0$

36·8

0·0009241 / 72 / 70 / 939

1367	2734
4221	1303
3010	7771
8598	8808
724	760

5092	5224	5353	5478	5599
9542	9542	9542	9542	9542
4634	4766	4895	5020	5141

$(C_2H_5)_2O$ — 48 / 58 / 16 / 74

0° ·00286	4564	1527	3037	51·5
2585	4125	1605	2520	50
2345	3701	16805	2020	51
212	3263	1753	1510	

20°

$\frac{0.00505}{0.00165} = 3.06$

0·001480
140
32
0·00165

7·04

8692	8692	8692	8692
4362	4518	4669	4814
3054	3210	3361	3506

$0.0_2 148026$ $0.0_5 3503$ $0.0_7 2701$

Propylide 109·5

60	487	6964	3758	3206	438
70	456	6590	38225	2768	426
80	4195	6227	3885	2342	51
90	387	5877	39455	1832	785
100	0·00369	5551	45045	1047	

$\frac{19}{60} = 32$ 86·8°

$\frac{0.0055}{0.00135} = 4.07$

9·36

2·2292	2292	2292	2292	2292
2·5224	5353	5478	5599	5717
7516	7045	7770	7891	9009

$0.0_2 1027$ $0.0_5 1866$ $-0.0_7 0005$

Äthylsulfid 90

50	0·00331	5198	2317	2881	443
60	3035	4821	2383	2438	43
70	279	4456	2447	2008	419
80	257	4099	2510	1589	412
90	237	3747	25705	1177	

$0.0_2 11964$ $0.0_5 18065$ $0.0_7 07882$

$\frac{1.2}{42} = 0.03$

69·7°

$\frac{0.00420}{0.00156} = 2.69$

6·19

Propionsäureanhydrid	90	0·004715	6735	3369	33[illegible]	47	
130	100	4295	6330	3428	290	38	
	110	399	6010	3485_5	252	50	46
		3595	5557	3541_5	202	42	
		371	5198	3596	160	39	120°
		306	4857	3649_5	121	38	
		284	4533	3701	83	37	$\frac{0{\cdot}0046}{146}=$
		263_5	4208	3752	46		

0·0010911 $\quad 0{\cdot}0_5 03829_5 \quad 0{\cdot}0_7 06515$

92·5

Para-Xylol	70	377	5793	2803	296	45	
106	80	345	5378	2865_5	251	42	
	90	317	5011	2926	209	42	91·7°
	100	282	4654	2985	167	40	
	110	270	4314	3042_5	127	38	
	120	250_5	3988	3098_5	89	37	
	130	233	3674	3153	52		

0·0097013 $\quad 0{\cdot}0_5 08714 \quad 0{\cdot}0_7 05287$

$\frac{0{\cdot}0042}{0{\cdot}0100} =$ ~~4·2~~

96·6

1139	1139	1139	1139	1139	1139	1139	1139
5599	5717	5832	5944	6053	6160	6263	6365
6738	6856	6971	7083	7192	7299	7402	7504

0253	0253	0253	0253	0253	0253	0253
5353	5478	5599	5717	5832	5944	6053
5606	5731	5852	5970	6085	6197	6306

0792	1584	0·001091	4.10^4
3010	4771	92	
5832	8139	281	·6628
9634	4494	1464	1644
9192	2814s		5984

9624	9248	1598	0·0097013
3010	4771	1334	~~[illegible]~~
9402	7232		1598
			1334
2036	1251		0·009995

Hg $\beta = 3.85 \cdot 10^{-6}$

0° $\alpha = 0.0001817$

C_6H_6

16	82
14.8	75
16	90
17.9	92
15.4	87
30.1:5	26:5 = 52

(16.02)

$\beta = 85.2 \cdot 10^{-6}$

$\alpha = 0.00123$

$a_{Hg} = \dfrac{273 \cdot \cancel{0.000}1817}{(13.6)^2 \quad 3.85 \cdot \cancel{10}}$

$\dfrac{289 \cdot \cancel{0.00}123}{\cancel{1.110} \cdot 8.52 \cdot 10^{-8}} \quad (0.863)^2$

12.16

22

1.0192

880

−317

863

4362
2594
6956
8525
8431
8525
9906

2670
5855
8525
2576

$a = 9786$

4609
0899
5508
8024
7484

9304
~~9360~~
8720
8024
6204

$a = \boxed{5602}$

$a\rho^2 = 18100$ atm

4173 atm

12.22
24
24
1.0264
208
858

273
22.75
295.75

$\varphi = \dfrac{a\rho}{RT}\mu$

9906
1335
1241
8715
2526

5705
+3010
8715

5602
9335
4937
3631
1306

4710
8921
3631

8921
7842
1303
9145

$0.135 \cdot 78^2$

$0.01789 \cdot 40000$

7156

φ dla temp. odpowiednij (?)

z równ. $\varphi = \dfrac{a\rho\mu}{RT}$

8213

C_6H_5OH $0.5\cancel{4}0$

24
16
6
40

$\rho = 0.0_4970(1 + 0.0_2 3177\, t + 0.0_4 550\, t^2)$

8021 6042
5021 7404
3042 3446

$\cancel{1}2015$ 2211 $\alpha =$
2211

$\beta = 1.4226 . 0.0_4 9\cancel{7}0$

80625

1530
9868 $\beta = 0.0_3 1380$ $\alpha = 0.001353$
1398 (1313)

9212 4624
8021 6042
0.07233 0666
6,50

$a = \dfrac{3364 . \cancel{0.00}1353}{(0.7522)^2 \cancel{\;} \cancel{0.000}138}$

0.05288 80625
116 05404
0.05404 ‖ 07522

$\varphi = \dfrac{13.53 \qquad .46}{1.38 .\ 0.7522 . \cancel{\;}}$

1313 1399
6628 8763
7941 0162
0162
7779

$\varphi = 5997$

	Hg	C_6H_6	C_2H_5OH
krit. T		288·5	243·6
π		47·89	62·76
d		0·3045	0·288
v			0·00713

$$\eta_1 : \eta_2 = \frac{\sqrt{m_1 T_1}}{\sqrt[3]{v_1}} : \frac{\sqrt{m_2 T_2}}{\sqrt[3]{v_2}}$$

C_6H_6 22·75

C_2H_5OH 63·40
273
330·40

243·6 288·5
273 273
516·6 561·5

5268
7494
2702
7131
5031

3657
273
92·7

Hg T_k = 302·2
Amylalk. 348·0
Äther 195
Äthylsulfid 284·67
Cam.Xylol 344·4

344·4 284·7 91·7
273 273 273
617·4 557·7 364·7
5371 7544 9624
4362 4362 4362
9733 2906 3986
8901
2893
9733
3159

~~207~~

5619
7464
3083
7906
5177

3294
273
56·4

377 32
325 28
317 — 25 — 26·5
292 22
270
250·5 19·5
491
364 37
331 33
303·5 27·5
 24·5
279 23·2
 22
257 20
237

$\frac{26\cdot0}{313}$ $\frac{1}{n}\frac{dn}{dt} \sim \frac{1}{617\cdot4}$

$\frac{233}{279}$ $\sim \frac{1}{557\cdot7}$

4150	3674	4150	3674
7906	7464	−4955	−4456
2056	1138	9195	9218
−4955	−4456	831	835
7101	7682		
513	586		

$\frac{1}{n}\frac{dn}{dt}$

	$p =$	$\mu =$			
CO_2: 5°	0·922	0·000925	9061	·04375	9224
	0·875	852	9304	04765 = 8828	
15	864	784	8943 − 05145 = 8429		
20	827	712	8525 − 0552 = 7973		
25°	783	625			
30°		539			

40
40

12
32
44

273
15
288

4594 + 0435 = 1029

4518 + 6435 = 0953

4669 + 6435 = 1104

~~96~~ 9440 + 6435 = 0875

$(C_2H_5)_2O$:						
74	15·8	0·002502				
	29·2	2180				
	47·2	1870	2718	1873	0845	43·9
	63·5	1626	2111	19805	0130	46·4
	78·7	1413	1501	20765	9425	~~39·6~~ 42·6
	100·4	1177	0708	2207	8501	

273
373·4

8692 − 5722 = 4414

8692

787 + 273 = 351·7

63·5 + 273 = 336·5

47·2 + 273 = 320·2

5461 − 8692 = 4153

8692 − 5269 = 3961

~~4718~~ 5054 − ~~2732~~ 3886

5054 − 8692 = 3746

$$(C_2H_5)_2O: \quad v = v_0 \left(1 + 0·0_2 1349 + 0·0_5 6554\,t^2 - 0·0_7 3449\,t^3 + 0·0_9 3377\,t^4\right)$$

~~996~~ ~~214~~

924

9657 − 3765 = 6292

7·05 / 15·2

8482 − 1818 = 6664

7·15 / 16·3

8543 − 2122 = 6421

$\frac{32·7}{42·6}$ = 5145 − 6294 = 8851

7·67

92·7

$\alpha_{10} = \frac{27}{5.913}$, $\frac{31}{5.880}$ } $\frac{29}{5.900} = \frac{58}{9000} = 0.00644$

9671	9342	9013	8684
1300	8165	5377	5285
0971	7507	4390	3969
1·1251	5632	2748	2499

2010	4771	6021
9671	9342	9013
8165	5377	5285
0846	9490	0319
1215	8892	1076

1·1251
563
249
1·2063
− 275
1·1788

$\alpha = $ 0·001349
1215
1076
0·003640
− 889
0·002751

4393
0719
3679

$\alpha = 233$ 0_{0}^{0}

$\frac{0080}{00644}$ $(\times 2\cdot 3)$ $\frac{00426}{00233}$ $(\times 2\cdot 3)$

= 9031
8089
0942

1·24

0294
9674
2620

1·83

$$\mu = \sqrt{m\theta}\;\psi\left(\frac{v}{b}\right)$$

$$\frac{d\mu}{d\theta} = \frac{1}{2}\sqrt{\frac{m}{\theta}}\;\psi + \sqrt{m\theta}\;\psi'\;\frac{1}{v}\frac{dv}{d\theta}\cdot\frac{v}{b}$$

$$\frac{d\mu}{dp} = \sqrt{m\theta}\;\psi'\;\frac{1}{v}\frac{dv}{dp}\;\frac{v}{b}$$

$$\frac{d\mu}{d\theta} = \frac{1}{2}\frac{\mu}{\theta} + \frac{d\mu}{dp}\cdot\frac{\frac{1}{v}\frac{dv}{d\theta}}{\frac{1}{v}\frac{dv}{dp}}$$

$$\frac{1}{\mu}\frac{d\mu}{d\theta} = \frac{1}{2\theta} + \frac{1}{\mu}\frac{d\mu}{dp}\cdot\frac{\frac{1}{v}\frac{dv}{d\theta}}{\frac{1}{v}\frac{dv}{dp}}$$

Experimental $\quad v = v_0(1 + 0{\cdot}0_2\,09003 + 0{\cdot}0_5\,19595\,t - 0{\cdot}0_7\,044998\,t^2)$

$$\alpha_{20} = 0{\cdot}0_2\,0973$$

μ Terpentinöl

20°	0·01461	1647	8353	68.43	~~70~~ 34.9
50°	968	9859	0141	1033	363
80°	716	8549	1451	13965	

7 14

$$\frac{1}{\mu} = a + bt$$

$$-\frac{1}{\mu^2}\frac{d\mu}{dt} = b = 34.2$$

$$\frac{1}{\mu}\frac{d\mu}{dt} = b\mu = \frac{0.4997}{30}$$

$$= 0.01666$$

5340
1647
6987

7°	0·001668
15°	0·001625
20°	0·00160 $= \frac{1}{\mu}\frac{d\mu}{dp}$ (later)

27

$\beta = 79.12 \cdot 10^{-6}$

$\varkappa_{20} = 0{\cdot}000973$

0·~~000~~973 . 0·0016

~~0·0000~~791

4362
2010
7372
2628

0·001831

1831

2041
9881
1922
− 8984
2938

0·01967

$$-\ 0{\cdot}0197$$
$$+\ 0{\cdot}0018$$
$$-0{\cdot}0167 \quad \| \quad -0{\cdot}0179 \quad \|$$

"beob" "ber."

$\eta = \eta_0 (1 + \alpha p)$

	CO_2	$(C_2H_5)_2O$	C_6H_6
	25·1°	209	209
$a \cdot 10^6 =$	7470	730	930
$\alpha =$	0·003040	0·000173	0·000091
		167	

638 Quincke
72 Rö
82 Jelemer
90 Amagat
75 de Metz
90 Vaglem

1

0·0906 . 585
5205
58
30
5385

0·001654

$\alpha =$ 0·001248

$\frac{0·000730 \cdot 0·001654}{0·000~~1~~74}$

$\frac{~~0·000~~930 \cdot ~~0·00~~1248}{~~0·0000~~91}$

8633
2186
0819
− 2227
8592

9685
0962
0647
− 9590
1057

4669
5331
101
232
1701

0·007232
171
−0·00552

0·01275
171
0·01104

~~8~~

0·0150
~~0·80~~

5° 0·00286
10° 0·002585
20° 0·002345
30° 0·00212

$\left.\begin{array}{l}\frac{240}{2465} \\ \frac{225}{2232}\end{array}\right\} =$ 0·0098

6.7	0.002668	4262	227 :	5.12	444
11.81	2532	4035	215	5.31	405
17.12	2410	3820	180	4.68	385
21.8	2312	3640	151	3.65	395
25.45	2233	3489	275	6.59	418
32.04	2096	3214			

3560	3324	2853	1790	4393
7093	7251	6702	5623	8189
6467	6673	5851	5967	6264

0°	0.009025	1435	110	0414	
100	759	118	704	8476	156
				1938	
20°	649	87	87	9395	— 150
30	562	70	6055	7822	144
40	492			1573	

Glycerin

	2.8	42.2	6253	3747	237
	8.1	25.2	4014	5986	397
	14.3	13.9	1430	8576	719s
	20.3	7.78	8910	1090	1285
	26.5	4.94	0937	3063	2024

3500
7243
6257

4123
7924
6199

2930
7924
5006

2239 : 5.3 = 422
2584 : 6.2 417
2520 : 6.0 420
1973 : 6.2 317

Imyl alk. ...

0·11129	04650	1758	119
7425	8707	1639	126
5091	7068	1513	118
3593	5555	1395	103
2606	4160	1292	
19355	2868	1189	103
1472	1679	1083	106
1147	0596	1010	73
909	9586	923	87
725	8663	845	78
608	7818	785	60
505	7033	708	77
429	6325		

$\mu = a e^{\alpha t + \beta t^2}$

$a = -\left(\dfrac{T \frac{\partial p}{\partial T}}{\rho^2 \frac{\partial \rho}{\partial p}}\right)_{p=0}$

$(C_2H_5)_2O$: $\beta_0 = 0·000146$ $\beta_{35} = 0·$ ~~...~~ 2277 || $\alpha_0 = 0·00148$

$\dfrac{T \frac{1}{v_0}\frac{\partial v}{\partial T}}{\frac{1}{v_0}\frac{\partial v}{\partial p}}$ $\frac{v}{v_0}$ v_0

			3010	4771	
5441	0882	6323	5441	0882	~~6323~~
1703	5445	4314	5445	4314	
7144	6327	0637	3896	9967	
1·0518	429	116	245	9925	

1·0518
43
12
1·0573

0·0014803
245
99
0·001825

$a = \frac{1}{\rho_0^2}\left\{\dfrac{273.\ 0·00148}{0·000146}\right\} \quad \Big\| \quad \left\{\dfrac{308.\ 0·001825\ .\ 1·057}{\cancel{1·057}\ 0·0002277}\right\}$

4362
1703
6065
− 1644
4421

= 2770

1886
2613
0241
1740
− 3573
1167

= 2610

$$p + \frac{a}{v^2} = \frac{R\theta\sqrt{2}}{v - v_0}$$

$$1 - \frac{2a}{v^3}\frac{\partial v}{\partial p} = -\frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial p}$$

$$-\frac{2a}{v^3}\frac{\partial v}{\partial \theta} = \frac{R\sqrt{2}}{v-v_0} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial \theta}$$

$$\boxed{\frac{\partial v}{\partial \theta} = -\frac{R\sqrt{2}}{v-v_0}\frac{\partial v}{\partial p}}$$

$$1 = \frac{\partial v}{\partial p}\left[\frac{2a}{v^3} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\right] \qquad = \frac{\frac{\partial v}{\partial p}\frac{R\sqrt{2}}{v-v_0}}{-\frac{\partial v}{\partial \theta}}$$

$$-\frac{R\sqrt{2}}{v-v_0} = \left[\frac{2a}{v^3} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\right]\frac{\partial v}{\partial \theta}$$

$$-\frac{2}{v}\frac{\partial v}{\partial \theta}\frac{R\theta\sqrt{2}}{v-v_0} = \frac{R\sqrt{2}}{v-v_0} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial \theta} \qquad \frac{a}{v^2} = \frac{R\sqrt{2}}{\frac{\partial v}{\partial \theta}}$$

$$-\frac{2}{v}\frac{\partial v}{\partial \theta}\theta = 1 - \frac{\theta}{v-v_0}\frac{\partial v}{\partial \theta}$$

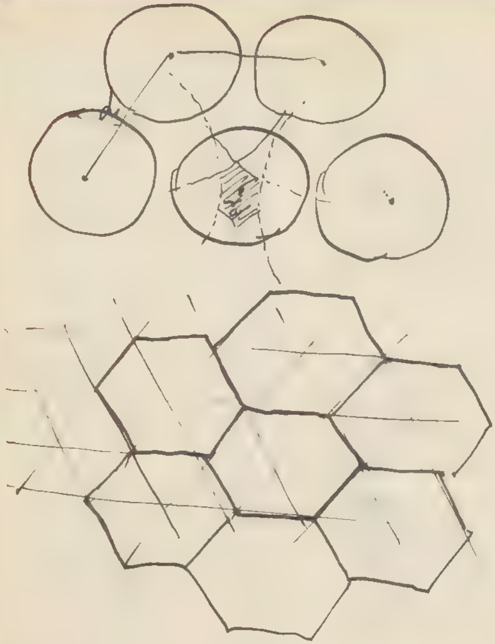
$$\theta\frac{1}{v}\frac{\partial v}{\partial \theta} = \frac{1}{\frac{1}{v-v_0} - \frac{2}{v}} = \frac{v(v-v_0)}{v-2(v-v_0)} = \frac{(v-v_0)}{2v_0 - v} \neq \frac{v-v_0}{v_0}$$

Dewar $\alpha_{20} = \frac{0{\cdot}00125 \, . \, 293}{0{\cdot}36} = \frac{v-v_0}{v_0}$

$$\frac{1}{v-v_0} \neq \frac{1}{\theta\frac{\partial v}{\partial \theta}}$$

$$\frac{\partial v}{\partial \theta} = -\frac{R\sqrt{2}}{\theta\frac{\partial v}{\partial \theta}}\frac{\partial v}{\partial p}$$

$$\frac{\partial v}{\partial p} = -\left(\frac{\partial v}{\partial \theta}\right)^2\frac{\theta}{R\sqrt{2}}$$



$$p+\frac{a}{v^2}=\frac{RT}{v_a}\left(\frac{a}{\frac{d}{2}}\right)^2$$

$$v_a : v=\left(\frac{a}{\frac{d}{2}}\right)^3$$

$$p+\frac{a}{v^2}=\frac{RT}{v}\;\frac{d}{2a}$$

$$=\cancel{\frac{RT}{v}\;\frac{d}{2(d-2b)}}$$

$$v_0 : v=(\sigma : d)^3$$

$$=(d-\cancel{a})^3 : a^3$$

$$\frac{v_0}{v}=1-\frac{3a}{d}$$

$$\frac{3a}{d}=1-\frac{v_0}{v}$$

$$p+\frac{a}{v^2}=\frac{3}{2}\frac{RT}{v}\frac{1}{\left(1-\frac{v_0}{v}\right)}=\frac{3}{2}\frac{RT}{v-v_0}$$

$\lim p=\infty$:

$$\cancel{\frac{\partial v}{\partial p}=-\frac{\frac{3}{2}RT}{p^2}}$$

$$v-v_0=\frac{\frac{3}{2}RT}{p+\frac{a}{v^2}}$$

$$\cancel{\frac{\partial v}{\partial p}=\frac{\frac{3}{2}RT}{\left(p+\frac{a}{v^2}\right)^2}\left(1-\frac{2a}{v^3}\frac{\partial v}{\partial p}\right)}$$

$$\lg(v-v_0)=\lg\left(\tfrac{3}{2}RT\right)-\lg\left(p+\tfrac{a}{v^2}\right)$$

$$\frac{1}{v-v_0}\frac{\partial v}{\partial p}=-\frac{1}{p+\frac{a}{v^2}}=-\frac{v-v_0}{\frac{3}{2}RT}\left(1-\frac{2a}{v^3}\frac{\partial v}{\partial p}\right)$$

$$\frac{\partial v}{\partial p}=-\frac{(v-v_0)^2}{\frac{3}{2}RT}=(v-v_0)\left(p+\frac{a}{v^2}\right)$$

$$=\frac{\frac{3}{2}RT}{\left(p+\frac{a}{v^2}\right)^2}$$

v_0 bedeutet nicht den thatsächlich von d. Mol. eingenommenen Raum, sondern den Raum d. Körpers bei grösster Compression.

$$\frac{\partial v}{\partial p} = \frac{1}{\frac{2a}{v^3} - \frac{3}{2}\frac{R\theta}{(v-v_0)^2}} = \frac{1}{\frac{2a}{v^3} - \left(p+\frac{a}{v^2}\right)^2 \cdot \frac{2}{3R\theta}}$$

$p_0 = 0.736$

$$R\theta = \frac{28}{74 \cdot 0.00125}$$

1.8692
0.0972 − 3
0.9664 − 2

1.4472
−0.9664 + 2
2.4808

$\frac{a}{v^2} = 2770$

3.4425
0.8669 − 1
3010
3.6104

6.8850
3010
71860
2.9579
42281

4771
2.4808
~~1.9963~~

0.00007793 — w rzeczywistości Amagat → 0.000107

16910
4077
12833

1083
8917

2770
3000
5770

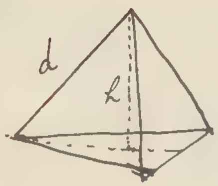
7612
5224
3010
8234
9579
8655

73400
408
−69320

8408
1592

→ 0.000032

0.00001443



$h = d\sqrt{\frac{2}{3}}$

Vollständige Übergangsfreiheit für horizontale Schichtenverschiebung wenn

$h \geqq 6$

Gar kein Übergangsfreiheit wenn

$h \leqq \frac{6}{2}\sqrt{3}$

4771
3010
1761
08805

$\cancel{\geq \frac{d}{6} \geq \sqrt{\frac{3}{2}}}$ Vollständig frei: $\frac{d}{6} > \sqrt{\frac{3}{2}} = 1{\cdot}225$

$\frac{v}{v_0} > 1{\cdot}84$

Mittelfrei: $\sqrt{\frac{3}{2}} > \frac{d}{6} > \frac{\sqrt{3}}{2} \cdot \sqrt{\frac{3}{2}}$

$\sqrt{\frac{3}{2}} > \frac{d}{6} > \frac{3}{2\sqrt{2}}$

Unfrei: $\frac{d}{6} < \frac{3}{2\sqrt{2}} = 1{\cdot}061$

$\frac{v}{v_0} < 1{\cdot}193$

4771
3010
1505
4515
0256
0708

Erster Körper: $\frac{d}{6} = 1{\cdot}061$

$v_0 : v = 6^3 : d^3$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial \theta} = \frac{R\varphi}{2a\rho - RT\left(\varphi + \frac{\rho}{\rho_0}\varphi'\right)}$$

$$= \frac{R\varphi}{2RT\varphi - \frac{2\mu}{\rho} - RT\varphi - RT\frac{\rho}{\rho_0}\varphi'}$$

$$= \frac{R\varphi}{RT\left(\varphi - \frac{\rho}{\rho_0}\varphi'\right) - \frac{2\mu}{\rho}}$$

dla $\mu = 0$:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial \theta} = \frac{\varphi}{T\left(\varphi - \frac{\rho}{\rho_0}\varphi'\right)} = \frac{1}{\theta}\,\chi\left(\frac{\rho}{\rho_0}\right)$$

Wielkości w odpowiadających sobie

[illegible]	99°	$\alpha = 0{\cdot}000182$!
C_6H_6 (~~[illegible]~~)	22·7°	$\alpha = 0{\cdot}00126$!
C_2H_5OH	63·4°	$0{\cdot}00135$
Dimethyläthylcarbinol	17·2°	$\alpha = 0{\cdot}00114$
Butylalk.	13·7°	$0{\cdot}00094$
Butylalk.	2·7°	~~$0{\cdot}000845$~~
$(C_2H_5)_2O$	20°	$\alpha = 0{\cdot}00165$
Paraxylol	92°	$\alpha = 0{\cdot}00100$

$$\frac{\partial\rho}{\partial p} = \frac{1}{RT(\varphi+\frac{\rho}{\rho_0}\varphi')-2a\rho} = \frac{1}{RT(\varphi+\frac{\rho}{\rho_0}\varphi'-2\varphi)-2\frac{p}{\rho}}$$

$$= \frac{-1}{RT(\varphi-\frac{\rho}{\rho_0}\varphi')+\frac{2p}{\rho}}$$

$$\left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_{p=0} = -\frac{1}{RT\rho(\varphi-\frac{\rho}{\rho_0}\varphi')}$$

$$\frac{\alpha}{\rho} = -\ \varphi R\rho$$

$$\mu = \cancel{n)m}\overbrace{(\lambda+\sigma)}^{d} \quad = \frac{Nc}{3} \text{ ilagosu}$$

$$p+\frac{a}{\rho^2} = (n)\ mc = R\theta\rho\varphi = \frac{3}{2}\frac{R\theta}{v-v_0}$$

$$\mu = \left(\frac{3}{2}\frac{R\theta}{v-v_0}\right)\frac{d}{c}$$

$$c = \sqrt{\frac{\theta}{m}}\cdot \gamma$$

Jäger Fortschr. p. 121

$$d = \sqrt[3]{\frac{m}{\rho}}\ \ldots \qquad d^3 = \frac{1}{\rho \underset{\text{anzahl in 1 gr}}{N}} = \frac{m}{\rho A} \qquad V = \frac{A}{m}$$

$$\mu = \frac{3}{2}\frac{R\theta}{v-v_0}\sqrt[3]{\frac{m}{\rho}}\sqrt{\frac{m}{\theta}} = \cancel{\ldots}$$

$$\sim \frac{\rho^2}{\sqrt{\theta}\cdot\rho^{1/3}}$$

$$\frac{1}{\mu}\frac{d\mu}{d\theta} = \frac{5}{3}\rho^{2/3}\frac{d\rho}{d\theta} - \frac{1}{2\theta} \qquad = -\frac{5}{3}\rho^{5/3}\cdot\alpha - \frac{1}{2\theta} \qquad !!!$$

$\cancel{\eta = \sqrt{m\theta}\,\psi\left(\frac{\beta}{b}\right)}$

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial t}\right)_{v=\text{const}} = \frac{1}{\eta}\frac{d\eta}{dt} - \frac{1}{\eta}\frac{\partial\eta}{\partial p}\frac{\alpha}{\beta}$$

$$= \cancel{\left(\frac{1}{\eta}\frac{\partial\eta}{\partial\theta}\frac{\partial\sigma}{\partial t}\right)_{v=\text{const}}}$$

$$\eta = \frac{\sqrt{m\theta}}{\sigma^2}\,\psi\left(\frac{v}{b}\right)$$

$b \sim \sigma^3$

$\frac{1}{b}\frac{\partial b}{\partial\theta} = 3\,\frac{1}{\sigma}\frac{\partial\sigma}{\partial\theta}$

$$\cancel{\frac{1}{\eta}\frac{\partial\eta}{\partial\theta} = -\frac{2}{\sigma} + \frac{\psi'}{\psi}\frac{1}{b}\frac{\partial v}{\partial\sigma}}$$

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial\theta}\right)_v = \frac{1}{2\theta} - \frac{2}{\sigma}\frac{\partial\sigma}{\partial\theta} - \frac{\psi'}{\psi}\frac{v}{b^2}\frac{\partial b}{\partial\theta}$$

$$= \frac{1}{2\theta} - \frac{1}{\sigma}\frac{\partial\sigma}{\partial\theta}\left[2 + 3\frac{\psi'}{\psi}\cdot\frac{v}{b}\right]$$

Pod założeniem, że σ zależy jedynie od θ, ale nie od v:

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial p}\right)_{t=\text{const}} = \frac{\psi'}{\psi}\frac{1}{b}\frac{\partial v}{\partial p} = -\frac{\psi'}{\psi}\frac{v}{b}\cdot\beta$$

	$(C_2H_5)_2O$	C_6H_6
$\frac{1}{\eta}\frac{\partial\eta}{\partial p} =$	0·000730	930
$\beta =$	0·000167	0·000091
$\frac{1}{\eta}\frac{d\eta}{dt} =$	−0·0098	−0·0150
$\frac{\alpha}{\beta}\frac{1}{\eta}\frac{\partial\eta}{\partial p} =$	0·00723	0·0127
$\frac{1}{\eta}\frac{\partial\eta}{\partial t} =$	−0·00157	−0·0023
zamiast	+0·00171	+0·00171
	328	401
$\frac{1}{\sigma}\frac{\partial\sigma}{\partial\theta} =$	−0·0003[illegible]	−0·00015

$\frac{\psi'}{\psi}\frac{v}{b} = -4{\cdot}3 \quad \| -10{\cdot}2$

186
79
205

274
1/096
3.84
8/22
29

Aluminium (Pionchon)

274) −250° 0° 250° 500° 625

0·1428 0·209 0·238 274 308

8'44

3'84

Naccari

Sb Behn

120) −133° 15° 100 200 300

0·0462 0·0489 0·0503 520 537

(N. Petterson)

Be 0–100 0–300

9.1 ~~0~~·0425 ~~0~~·0506

Pb −90° 15° 100 200 300

207 0·0294 0·02993 31~~0~~ 324 338

H F Weber

B −40° 27° 126 233

11 0·1915 238 307 366

Cd −133° 21° 300°

112

0·0498 0·0551 0·0617

Cr −200 −100 0° 200 400 600

52·1

0·0666 0·0898 0·104 118 133 187

Schmitz Mickel Pionchon

Fe −760 60° 157 250 350 860 1100

55·9 0·0890 0·119 0·1275 176 324 218 199

Se	50°	220°				
72.5	0.0737	0.0757				
Au	50°	0.0316				
197						
Ir	−84°	50	700			
193	0.0282	0.0323	0.0401			
K	−25°	0.1662				
39.2						

Fihl

Co	−84°	57°	83	282	322
59	0.0822	103	109	121	123

Abondan

250	400	500
145	184	204

C	−50.5°	10.7	140	247	607	985
Graphit 12	0.0635	113	222	303	441	459

Pt	-84°	59°	500	1000	1500
195	0·0293	324	377	409	461

Prędkość, z którą się kurczy przerwana [illegible]

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= \alpha \\ m &= \delta\rho x \end{aligned}\right\} \quad x\frac{d^2x}{dt^2} = \frac{\alpha}{\delta\rho}$$

$$m\frac{d^2x}{dt^2} = -kx - ax^3$$

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 = c - \frac{k}{2}x^2 - \frac{a}{4}x^4 = c - U_x$$

$$\frac{\sqrt{m}\,dx}{\sqrt{c-kx^2-\frac{ax^4}{2}}} = dt$$

$$\frac{1}{\sqrt{c-kx^2-\frac{ax^4}{2}}} = \frac{1}{\sqrt{c-kx^2}}\left[1-\frac{ax^4}{2(c-kx^2)}\right]^{-\frac{1}{2}}$$

$$= \cancel{\sqrt{c-kx^2}} - \frac{ax^4}{4\sqrt{c-kx^2}}$$

$$= \frac{1}{\sqrt{c-kx^2}} + \frac{ax^4}{4\sqrt{c-kx^2}^{\,3}}$$

$$\sqrt{c-kx^2-\frac{ax^4}{2}} = \sqrt{c-kx^2} - \frac{ax^4}{4\sqrt{c-kx^2}}$$

$$\frac{1}{t}\int U_x\,dt = \frac{1}{t}\left[U_x t - \int dt\, t\,\frac{\partial U}{\partial x}\frac{\partial x}{\partial t}\right]$$

$$\cancel{\frac{dx}{\sqrt{1-\alpha x^2}}} \qquad \cancel{\frac{dx}{\sqrt{1+\frac{\alpha^2}{4\beta}} - \left(\frac{\alpha}{2\sqrt{\beta}}+x^2\sqrt{\beta}\right)^2}}$$

$$= \frac{1}{2t}\int\frac{c-kx^2-\frac{ax^4}{4}}{\sqrt{c-kx^2-\frac{ax^4}{2}}}\sqrt{m}\,dx$$

$$= \frac{\sqrt{m}}{2t}\int_0 \sqrt{c-kx^2-\frac{ax^4}{2}}\,dx$$

$$c-kx^2-\frac{ax^4}{4}=0$$

$$\cancel{\int\frac{dx}{\sqrt{c-kx^2}} = \sqrt{\tfrac{1}{k}}\arcsin x\sqrt{\tfrac{k}{c}}}$$

$$\int\frac{dx}{\sqrt{1-\alpha^2x^2}} = \frac{1}{\alpha}\arcsin\alpha x$$

$$-\int\frac{\alpha x^2}{\sqrt{}^{\,3}}\,dx = -\frac{1}{\alpha^2}\arcsin\alpha x + \frac{1}{\alpha}\frac{x}{\sqrt{1-\alpha^2x^2}}$$

$$\int\sqrt{1-\alpha^2x^2}\,dx = \int\frac{1}{\sqrt{}} - \alpha^2\int\frac{x^2}{\sqrt{}}\,dx = x\sqrt{1-\alpha^2x^2} + \int\frac{x^2\alpha^2}{\sqrt{}}$$

$$\cancel{J = \tfrac{1}{\alpha}\arcsin\alpha x}$$

$$\int\frac{\alpha^2x^2}{\sqrt{}} = \frac{1}{2}\left[\frac{1}{\alpha}\arcsin\alpha x - x\sqrt{1-\alpha^2x^2}\right]$$

$$J = \cancel{\frac{1}{\alpha}\arcsin\alpha x} + \alpha\frac{\partial J}{\partial\alpha}$$

$$\int\sqrt{1-\cdots} = \frac{1}{2}\left[\frac{1}{\alpha}\arcsin\alpha x + x\sqrt{1-\alpha^2x^2}\right]$$

$$\frac{1}{2}\left(\cancel{\frac{\alpha}{\alpha\sqrt{1-\cdots}}} + \frac{1}{\sqrt{}} - \frac{\alpha^2x^2}{\sqrt{}}\right) = \frac{1}{2}\left(\frac{1-\alpha^2x^2+1-\alpha^2x^2}{\sqrt{}}\right)$$

$$\frac{d}{dx}\left(x^3\sqrt{1-\alpha^2x^2}\right) = 3x^2\sqrt{1-\alpha^2x^2} - \alpha^2x^2\sqrt{1-\alpha^2x^2}$$

$$\frac{d}{dx}\left(x\sqrt{1-\alpha^2x^2}\right) = 1\cdot\sqrt{1-\alpha^2x^2} - \alpha^2x^2\sqrt{1-\alpha^2x^2} \quad \Big|\, 3$$

$$\frac{d}{dx}(\alpha^2x^3+3x)\sqrt{1-\alpha^2x^2} = 3\sqrt{1-\alpha^2x^2} - \alpha^4x^4\sqrt{1-\alpha^2x^2}$$

$$\alpha^4\int x^4\sqrt{1-\alpha^2x^2}\,dx = \frac{3}{2}\left[\frac{1}{\alpha}\arcsin\alpha x + x\sqrt{1-\alpha^2x^2}\right] - (\alpha^2x^3+3x)\sqrt{1-\alpha^2x^2}$$

$$\int x^4\sqrt{1-\alpha^2x^2}\,dx = \frac{3}{2\alpha^5}\arcsin\alpha x - \left(\frac{x^3}{\alpha^2} + \frac{3x}{2\alpha^4}\right)\sqrt{1-\alpha^2x^2}$$

$$\cancel{\sqrt{c-kx^2-\frac{\alpha x^4}{2}} = \left[(c-kx^2)\left(1-\frac{\alpha x^4}{2(c-kx^2)}\right)\right]^{\frac{1}{2}} =}$$

$$\frac{d}{dx}\frac{x^3}{\sqrt{}} = \frac{3x^2}{\sqrt{}} + \frac{\alpha^2x^4}{\sqrt{}}$$

$$\alpha^4\int\frac{x^4}{\sqrt{1-\alpha^2x^2}}dx - 3\int\frac{dx}{\sqrt{}} =$$

$$= \frac{\alpha^2x^3}{\sqrt{}} - \frac{3x}{\sqrt{}}$$

$$\frac{d}{dx}\frac{x}{\sqrt{1-\alpha^2x^2}} = \frac{1}{\sqrt{}} + \frac{\alpha^2x^2}{\sqrt{}}$$

$$= x\frac{\alpha^2x^2-1}{\sqrt{}} = -x\sqrt{1-\alpha^2x^2}$$

$$- \frac{2x}{\sqrt{1-\alpha^2x^2}}$$

$$\alpha^4\int\frac{x^4}{\sqrt{}} = -\frac{3}{\alpha}\arcsin\alpha x + x\sqrt{1-\alpha^2x^2} + \frac{2x}{\sqrt{}}$$

$$\frac{3}{\sqrt{1-\alpha^2x^2}} - \sqrt{1-\alpha^2x^2} + \frac{\alpha^2x^2}{\sqrt{}} = \frac{\cancel{3-2} + \alpha^2x^2 + \alpha^2x^2}{\sqrt{}} - \frac{2\alpha^2x^2}{\sqrt{}^3}$$

$$-\frac{2}{\sqrt{}} + \qquad = \frac{2\alpha^2x^2}{\sqrt{}^3}\left[1-(1-\alpha^2x^2)\right]$$

$$\int_0^{\frac{1}{\alpha}}\frac{x^4\,dx}{\sqrt{1-\alpha^2x^2}} = -\frac{3}{\alpha^5}\arcsin\alpha x + \frac{x}{\alpha^4}\sqrt{1-\alpha^2x^2} + \frac{2x}{\alpha^4\sqrt{\cdots}}$$

$$= -\frac{3\pi}{2\alpha^5}$$

$$\infty$$

$$\overline{U}_{xa} < \overline{U}_{x0}$$

$$m\frac{d^2x}{dt^2} = -k(x-x_0) - f(x) = -\frac{\partial U}{\partial x}$$

$$-\overline{U} = \overline{\frac{mc^2}{2}}$$

~~jirili f(x) = punt~~

lab $f(x) = \cancel{f(x_0) + (x-x_0) f'(x_0)}$

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 = \underbrace{-\frac{k}{2}(x-x_0)^2 - \int f(x)\,dx}_{-U \quad + c} = -\frac{k}{2}(x-x_0^2) - \left[x f(x_0) + \frac{(x-x_0)^2}{2} f'(x_0)\right] + c$$

$$\overline{U} = -\frac{1}{t}\int \frac{m}{2}\left(\frac{dx}{dt}\right)^2 dt$$

$$= -\frac{1}{t}\int U\,dt = -\frac{1}{t}\int U \frac{dx\sqrt{\frac{m}{2}}}{\sqrt{c-U}}$$

$$\overline{\frac{m}{2}\left(\frac{dx}{dt}\right)^2} = c - \overline{U} = \theta$$

$$c + \frac{1}{t}\int_0^c U \frac{dx\sqrt{\frac{m}{2}}}{\sqrt{c-U}} = \theta = c + \frac{\int_0^c U \frac{dx}{\sqrt{c-U}}}{\int_0^c \frac{dx}{\sqrt{c-U}}} =$$

$$\frac{m}{2} b^2\alpha^2 = -\frac{k}{2}a^2 - (x_0+a) f(x_0) - \frac{a^2}{2} f'(x_0) + c$$

$$c = \frac{m}{2} b^2\alpha^2 + \frac{k+f'(x_0)}{2} a^2 + (x_0+a) f(x_0)$$

$$-\overline{U} = \overline{\frac{m}{2}\left(\frac{dx}{dt}\right)^2} - c$$

$$\frac{m}{2}\left(\frac{d\xi}{dt}\right)^2 = -\frac{k}{2}\xi^2 - \xi f(x_0) + \frac{\xi^2}{2} f'(x) + c'$$

$$x - x_0 = \xi = a + b\sin\alpha t$$

$$-mb\alpha^2\sin\alpha t = -ka - kb\sin\alpha t - f(x_0) - (a + b\sin\alpha t) f'(x_0)$$

$$-mb\alpha^2 = -kb - bf'(x_0) \qquad ka + f(x_0) + af'(x_0) = 0$$

$$\alpha = \sqrt{\frac{k - f'(x_0)}{m}} \qquad a = -\frac{f(x_0)}{k + f'(x_0)}$$

$$\left(\frac{d\xi}{dt}\right)^2 = b^2\alpha^2\cos^2\alpha t$$

$$\overline{\frac{m}{2}\left(\frac{d\xi}{dt}\right)^2} = \frac{mb^2\alpha^2}{4} = \frac{b^2}{4}\left(k - f'(x_0)\right) = \kappa\theta$$

$$\cancel{\frac{m}{2} b^2\alpha^2\cos^2\alpha t = -\frac{k+f'(x_0)}{2}\left[a^2 + b^2\sin^2\alpha t + 2ab\sin\alpha t\right] - \left[a + b\sin\alpha t\right] f(x_0) + c'}$$

$t=0$: $$\frac{m}{2} b^2\alpha^2 = -\frac{k+f'(x_0)}{2} a^2 - a f(x_0) + c'$$

$$\bar{U} = \frac{mb^2\alpha^2}{4} - \frac{f(x_0)^2}{2(k+f'(x_0))} + x_0 f(x_0)$$

W razie jeśli $f(x_0) = f'(x_0) = 0$ $\qquad \bar{U} = \bar{T} = \kappa\theta$

W przeciwnym razie różnica

którego $f(x_0) > 0$ (siła przyciąg.)

$f'(x_0) < 0$ ale malejąca zależn.

może być większa lub mniejsza

ale $\frac{\partial U}{\partial \theta}$ niezmieniona

$$m\frac{d^2x}{dt^2} = -C + kx$$

Kropelka elipsoidalna.

$ds_1 : ds_2$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{y\,dy}{x\,dx} = -\frac{b^2}{a^2}$$

$$ds = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= dx\sqrt{1+\left(\frac{b^2x}{a^2y}\right)^2}$$

$$F = 4\pi\int_0^a x\,dx\sqrt{1+\frac{b^4x^2}{a^4y^2}}$$

$$= 4\pi\int_0^b y\,dy\,\frac{a^2}{b^2}\sqrt{1+\frac{b^2}{a^4y^2}\left(a^2b^2 - a^2y^2\right)}$$

$$= 4\pi\frac{a}{b^2}\int dy\sqrt{a^2y^2 + b^4 - b^2y^2} = \frac{4\pi a}{b^2}\int dy\sqrt{b^4 + (a^2-b^2)y^2}$$

$$= 4\pi a\int dy\sqrt{1+\frac{a^2-b^2}{b^4}y^2}$$

$$\int\sqrt{1+\alpha x^2}\,dx = x\sqrt{1+\alpha x^2} - \underbrace{\int\frac{\alpha x^2}{\sqrt{1+\alpha x^2}}\,dx}$$

$$+\int\left\{\frac{1}{\sqrt{1+\alpha x^2}} - \sqrt{1+\alpha x^2}\right\}dx$$

$$\cancel{\frac{\sqrt{1+\alpha x^2}}{2} - \frac{\alpha x^2}{2\sqrt{}} + \frac{1}{2\sqrt{\alpha}}}\ \frac{\sqrt{\alpha}}{\sqrt{1-\alpha x^2}}$$

$$= \frac{x\sqrt{1+\alpha x^2}}{2} + \cancel{\frac{1}{2\sqrt{\alpha}}\ldots}$$

$$+ \frac{1}{2\sqrt{\alpha}}\lg\left(x\sqrt{\alpha} + \sqrt{1+\alpha x^2}\right)$$

$$= 4\pi a\left\{\frac{b\sqrt{1+\varepsilon^2}}{2} + \frac{b}{2\varepsilon}\lg\left(\varepsilon + \sqrt{1+\varepsilon^2}\right)\right\}$$

$$= 2\pi ab\left\{1 + \frac{\varepsilon^2}{2} + \frac{1}{\varepsilon}\left(\varepsilon + \frac{\varepsilon^3}{2} - \frac{\varepsilon^3}{2}\right)\right\}$$

$F \neq 4\pi a b \left(1 + \frac{a^2 - b^2}{6b^2}\right)$?

$\frac{a}{b} = \kappa$

$\neq 4ab\pi = 4\kappa b^2 \pi = 4\pi\kappa \left(\frac{c}{\kappa}\right)^{2/3} = 4\pi c^{2/3} \kappa^{1/3}$

$a b^2 = c = \kappa b^3$

$\kappa = 1 + \alpha$

$F = \frac{4\pi c}{b}$

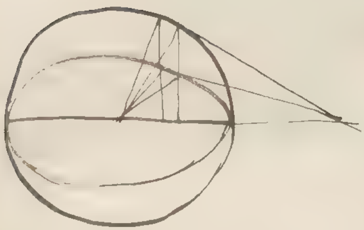
$\Delta F = 4\pi c^{2/3}(\kappa^{1/3} - 1)$

$= \frac{F_0 \alpha}{3}$

$=$

$y^2 = \xi r \cos\varphi = r^2 \sin^2\varphi$

$\xi = \frac{r \sin^2\varphi}{\cos\varphi}$



$ds : r d\varphi = \sqrt{\xi^2 + \alpha^2 y^2} : \sqrt{\xi^2 + y^2}$

$= \sqrt{\frac{\sin^4\varphi}{\cos^2\varphi} + \alpha^2 \sin^4\varphi} : \sqrt{\frac{\sin^4\varphi}{\cos^2\varphi} + \sin^4\varphi}$

$= \sqrt{1 + \alpha^2 \cos^2\varphi} : \sqrt{1 + \cos^2\varphi}$

Według Einsteina p 374:

$$\sqrt{\overline{x^2}} = \sqrt{\frac{\int_{-\infty}^{+\infty} x^2 e^{-\left(\frac{N}{RT}\right)U} dx}{\int e^{-\left(\frac{N}{RT}\right)U} dx}}$$

$$\frac{N}{RT} = h$$

$$v = q\,x$$

Gaz w walcowatym naczyniu, tłok równoważący ciężarem swoim ciśnienie gazu

$$P = pq$$

$$U = pq\,x$$

$$\frac{\int_0^\infty x^2 e^{-(hpq)x} dx}{\int_0^\infty e^{-(hpq)x} dx} = \frac{\frac{2}{\alpha^3}}{\frac{1}{\alpha}} = \frac{2}{\alpha^2}, \qquad hpq = \alpha$$

$$\cancel{\int x^2 e^{-\alpha x} dx = \frac{e^{-\alpha x}}{\alpha} x^2 - 2\int x e^{-\alpha x} dx}$$

$$\int_0^\infty e^{-\alpha x} = \frac{1}{\alpha}$$

$$\int_0^\infty x e^{-\alpha x} = \frac{1}{\alpha^2}$$

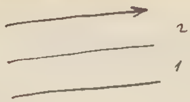
$$\int x^2 e^{-\alpha x} = \frac{2}{\alpha^3}$$

$$\sqrt{\overline{x}^2} = \frac{\sqrt{2}}{\alpha}$$

$$\Delta x = \frac{\sqrt{2}}{hpq}$$

$$\frac{\Delta x}{x} = \frac{\sqrt{2}}{phv}$$

$$\int \mu \left(\frac{du}{dy}\right)^2 dy$$



$$u_1 = \alpha_1 y$$

$$u_2 = \alpha_2 y + \beta$$

$$\frac{\alpha_1}{2} = \frac{\alpha_2}{2} + \beta$$

$$u_2 = \alpha_2 y + \frac{\alpha_1 - \alpha_2}{2}$$

$$\alpha_2 + \frac{\alpha_1 - \alpha_2}{2} = u$$

$$\frac{\alpha_2 + \alpha_1}{2} = u$$

$$\mu_1 \alpha_1 = \mu_2 \alpha_2$$

$$\mu_1 (2u - \alpha_2) = \mu_2 \alpha_2$$

$$\alpha_2 = \frac{2\mu_1 u}{\mu_1 + \mu_2} \qquad \alpha_1 = \frac{2\mu_2 u}{\mu_1 + \mu_2}$$

$$\beta = \frac{(\mu_2 - \mu_1) u}{\mu_1 + \mu_2}$$

$$\int \mu_1 \alpha_1^2 + \mu_2 \alpha_2^2$$

$$\frac{1}{2} \frac{4u^2}{(\mu_1 + \mu_2)^2} (\mu_1 \mu_2^2 + \mu_2 \mu_1^2) = 2u^2 \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$$

$$\frac{2u^2 (\mu - d\mu)(\mu + d\mu)}{2\mu} = u^2 \frac{\mu^2 - d\mu^2}{\mu}$$

[illegible]

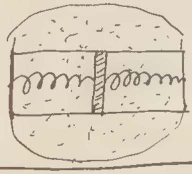
I). Tłok w naczyniu walcowatem zamkniętem obustronnie, położony w samym środku

Będzie wykonywać ruchy na prawo i lewo średnio dochodzące z pozycji równowagi?

II). Zastępujemy gaz z jednej strony przez sprężynę działającą siłą p proporc. do $\frac{1}{x}$.

$px = \text{const.}$ (izoterm.?) lub ogólnie do odpowiedniej wielkości p dla zgęsz. x

III). Tak samo z drugiej strony, a gazowi zostawiamy wolny przystęp



Przypadek III da się obliczyć według Einsteina

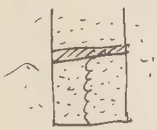
Czy I i III są identyczne?

Albo łatwiej?

I). Tłok o ciężarze $= p\,q$

odgraniczający szczelnie dolny gaz

III).



I). Tłok z przyczepioną siłą $(p_0 - p v_0)$ równoważącą właśnie ciśnienie

Wtedy każda pozycja równie możliwa $dW_1 = A\,dv$

II). Tłok bez tych sił przyczepionych

Wtedy $dW_2 : dW_1 = 1 : e^{-hU}$

$$dW_2 = A\,dv\,e^{-hU} = A\,dv\,e^{-h\int(p_0 - p v_0)\,dv}$$

---

zamiast x można wziąć: $V = vq$

długość taka że na cm² pokrywy być v

Siła przy przesunięciu o dv: $= 2\,\frac{dp}{dv}\cdot q$

$$\sqrt{\overline{v^2}} = \sqrt{\frac{RT}{2N\frac{dp}{dv}q}}$$

$$\frac{N}{RT} = h$$

$$\frac{p\mu}{\rho T} = R$$

$$\frac{10^6\cdot 30}{273\cdot 0.0013} = 10^8$$

$N = 4\cdot 10^{23}$

$$\sqrt{\frac{p\mu\,T}{\rho T\;2N\frac{dp}{dv}q}} = \sqrt{\frac{p}{2n_0\frac{dp}{dv}q}}$$

$$= \sqrt{\frac{pv}{2n_0 V\frac{dp}{dv}}}$$

$qv = V$

$p = \frac{RT}{v}$

$\frac{dp}{dv} = - \frac{RT}{v^2}$

$\Delta v = \sqrt{\frac{p v \cdot v^2}{2 n_0 V\ RT}}$ ~~itd~~

$\frac{\Delta v}{v} = \frac{1}{\sqrt{2 n_0 V}} = \frac{1}{\sqrt{2 \nu}}$

podraz przy oblicz. Boltzm-Einstein. $\frac{1}{\sqrt{2\nu\pi}}$

(różnica w $\overline{\Delta v}$ i $\sqrt{\overline{\Delta v^2}}$)

Ogólnie zatem

$$\frac{\Delta v}{v} = \sqrt{\frac{RT}{2 \nu v^2 \frac{dp}{dv}}} = \sqrt{\frac{\frac{p}{v} \frac{dv}{dp}}{2\nu}}$$

Odwrotnie argumentując:

prawdop. zboczenia dv : $A e^{-\left(\frac{N}{RT} \frac{dp}{dv} q\right) v^2} dv$

Spytajmy jaką substytuować należy zamiast p
idealną

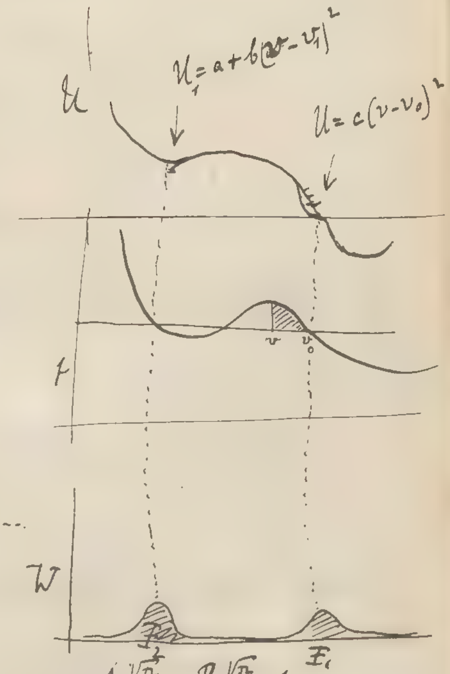
$f = q(p_v - p_{v_0})$

energia pot. ny. $U = q\left[\int_{v_0}^{v} p_v\, dv - \cancel{\ } p_{v_0} (v - v_0)\right]$ $\underbrace{\qquad\qquad}_{= \text{powierzchnia}}$

$dW = A\, e^{+hU} dv$ $\Big\|$ $= q\left[dv \left(\frac{\partial U}{\partial v}\right) + \frac{dv^2}{2} \left(\frac{\partial^2 U}{\partial v^2}\right) \ldots\right]_{v_0}$

$= q\left[dv \cdot p_v + \int_{v_0} \frac{\partial p}{\partial v} dv - p_0\right]$ $= \left(\frac{\partial p}{\partial v}\right)_0 dv^2 + \ldots$

$\frac{dv^2}{2} \left(\frac{\partial p}{\partial v}\right)_0 + \ldots$



W obu miejscach krzywa prawdop. W ma charakter

krzywej ~~[illegible]~~ $A e^{-\alpha v^2} dv$ $\qquad B e^{-\beta v^2} dv$

$A\sqrt{\frac{\pi}{\alpha}} + B\sqrt{\frac{\pi}{\beta}} = 1$

całk. powierzchnia $\int_{-\infty}^{+\infty} A e^{-\alpha v^2} dv = A\sqrt{\frac{\pi}{\alpha}}$ $\qquad$ przyczem $\alpha = \frac{hU}{\Delta v^2}$

~~$E_1 = E_2$ gdy $U_1 = U_2$~~ $\qquad$ więc powierzchnie $E_1 = E_2$

jeśli $U = 0$

Rayleigh I p. 108   IV p. 400

$d(U+I) + Ap\,dv = c_v dT + AT\left(\frac{\partial p}{\partial T}\right) dv$

$\frac{\partial U}{\partial v} + \frac{\partial I}{\partial v} + Ap = AT\left(\frac{\partial p}{\partial T}\right)$

$\cancel{\frac{\partial U}{\partial T}} + \frac{\partial I}{\partial T} = c$

$T_3 \frac{K'-K}{K'+2K}$ $K = n^2$ $n = n_0(1+\Delta)$

$$3\,\frac{n^2-n_0^2}{n^2+2n_0^2} = \frac{2\Delta + \Delta^2}{1+\frac{2}{3}\Delta+\frac{\Delta^2}{3}}$$

$(1+x)^{-1} = 1 - x + x^2 - \dots$

$$= 2\Delta\underbrace{\left(1+\frac{\Delta}{2}\right)\left(1+\frac{2\Delta}{3}+\frac{\Delta^2}{3}\right)^{-1}}_{\underbrace{1+\frac{\Delta}{2} - \frac{2}{3}\Delta\left(1+\frac{\Delta}{2}\right) - \frac{\Delta^2}{3}\left(1+\frac{\Delta}{2}\right) + \left(\frac{2\Delta}{3}\right)^2\left(1+\frac{\Delta}{2}\right)}}$$

$$= 1+\frac{\Delta}{2} - \frac{2\Delta}{3} - \frac{\Delta^2}{3} - \frac{\Delta^2}{3} + \frac{4\Delta^2}{9}$$

$$= 1 - \frac{\Delta}{6}$$

$\mu' = 1 + \Delta$

$$= 2\Delta - \frac{\Delta^2}{3}$$

$$\sum{}' T\left(2\Delta - \frac{\Delta^2}{3}\right)$$

$$h = 24\pi^3 n \frac{T^2}{\lambda^4} \frac{(\mu'^2-1)^2}{(\mu'^2+2)^2} = \frac{32}{3}\pi^3 n \frac{T^2}{\lambda^4}\Delta^2$$

coefficient of opacity

$$= \frac{32\pi^3(\mu-1)^2}{3n\lambda^4}$$ μ = refr. ind. of composed medium

[termodynam. Entropia określona tylko w razie równowagi ?]

Jeśli Entropia = lg[Wahrscheinl. of ~~e~~] to należałoby oczekiwać ogólnie:

$dW = A\,e^{-\alpha S}\,dv$?

$dW = dW_0\,e^{-\alpha(S-S_0)}$ izoterm.:

$\Delta\varphi =$

$$= dW_0\,e^{-\alpha\frac{\Delta\varphi}{T}} = dW_0\,e^{\underbrace{-\frac{\alpha}{T}\left(\Delta U + \int_{v_0}^{v} p\,dv\right)}}$$

niepewne, bo to tylko ważne dla ~~..~~ normalnych stanów równowagi

Próba: gaz idealny $\Delta U = 0$

$$\int_{v_0}^{v} p\,dv = RT \lg\frac{v}{v_0} = RT\lg(1+\delta) = RT\delta$$

$$dW_0\, e^{-\alpha R\delta}$$ stimmt nicht!

~~Gleichförmige Verteilung ist die wahrscheinlichste insofern jedes Volumelement ∞ viel Mol. enthält, sonst bei Endlichkeit der enthaltenen Mol.-Zahl treten die Ungleichförmigkeiten auf. Hängt das mit der Ersetzung von ... zusammen?~~

Termodynamicznie nie można ΔS określić, bo stan zboczenia przy δ, nie jest stanem równowagi termodyn.

Można tak argumentować:

Gaz zamknięty tłokiem na który z zewnątrz spoczywa normalne ciśnienie p_0 przyjmuje samodzielnie anormalną objętość v; jaka różnica Entropii? $\Delta S = \frac{\delta\varphi}{T}$

Ten sam stan można było osiągnąć wykonując pracę (dodatkową) przy normalnych warunkach

$$\int p\,dv - p_0(v - v_0)$$

Więc: $\delta\varphi = \Delta U + \int p\,dv - p_0(v-v_0)$

Toby się zgadzało z wzorem dla gazów idealnych

ale różnica ogólna ΔU

Zarzuty: spoczywa

a) $\Delta S = \frac{\delta\varphi}{T}$ ważne tylko dla procesów odwracalnych

Izoterma krytyczna $\frac{\Delta p}{p_k} = -\frac{3}{2}\left(\frac{\Delta v}{v_k}\right)^3$ według VdW

$$\Delta p = \frac{3}{2} p_k \frac{\Delta v^3}{v_k^3}$$

$$\int_{\Delta v = 0}^{\Delta v} \Delta p \, dv = -\frac{3}{8} p_k \frac{\Delta v^4}{v_k^3}$$

$$\frac{3}{8} p_k \frac{\Delta v^4}{v_k^3}$$

4.10^{19}

$$A e^{-\frac{N}{RT} \cdot q \frac{3}{8} p_k \frac{\Delta v^4}{v_k^3}} =$$

$$A e^{-\frac{N}{RT} \frac{3}{8} \frac{V}{v} p_k v_k \left(\frac{\Delta v}{v}\right)^4}$$

$$p_k v_k = R T_k \cdot \frac{3}{8}$$

$$A e^{-\frac{9}{64} \frac{T_k}{T} \delta^4}$$

$$\sqrt{l} = \lambda \frac{3.10^5}{6.10^{-4}}$$

$= 1.20$

$l = 400$

$$\bar{\delta} = \sqrt[4]{\frac{64}{9} \frac{T}{T_k\, v}}$$

$v = (0.2/\mu)^3 = 5.10^5$ $\quad \delta = 10^{-1}$ i.!!

$$n - 1 = \alpha v$$

$$dn = \alpha\, dv$$

$$= \alpha v \delta$$

$$\frac{dn}{n-1} = \delta$$

$$dn = (n-1)\delta$$

$$\frac{dn}{n} = \sqrt{} \frac{n-1}{n} \cdot \delta$$

$$N \pi 6^2 = \frac{1.25}{\sqrt{2} \cdot \lambda}$$

$$(n-1) \frac{2\sqrt{2}}{\sqrt{\pi}} \sqrt{\frac{l \sqrt{2}}{\lambda}}$$

$$= (n-1) \sqrt{\frac{8\sqrt{2}}{\pi} \frac{l \lambda}{1.25}}$$

$$(n-1) l \delta < \frac{\lambda}{2} = (n-1) \frac{l}{\sqrt{2\pi \cdot 4.10^{19} \cdot \pi^2 \pi l}}$$

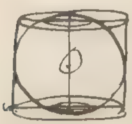
$$3.10^{-5} = 3.10^{-4} \frac{\sqrt{l}}{\pi \pi \sqrt{10^{20}}}$$

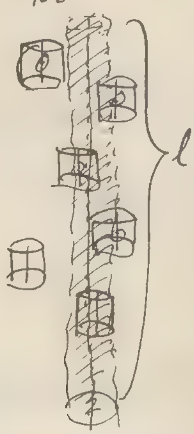
$$\sqrt{l} = 10^{-1} \cdot \pi \cdot 10^{-8} \sqrt{10^{20}} = 10.\pi = 31$$

$l = 900$ cm

$\delta =$

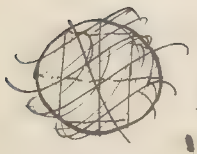
Promień przechodzący przez gaz, zastępuję każdą kulę przez walec o równej objętości i równym przekroju: πr^2





Tyle walców zostanie przeciętych ile punktów O się znajduje w obrębie walca o przekroju $\pi \varepsilon^2$ wykreślonego koło promienia jako osi więc w obrębie objętości $v = \pi r^2 l$

Powierzchnia zboczenia od normalnej objętości optycznej nl będzie zatem określone pierwiastkiem zboczenia od gęstości normalnej w obrębie v



$2r = 1\mu\mu$

$r = 0{\cdot}5 . 10^{-7}$ cm | or from optical measurements $0{\cdot}1 . 10^{-7}$ cm ?

$v = \frac{\pi}{4} . 10^{-14} l$

Terpentyna

$l = 250$ cm

$\nu = n v = \frac{\pi}{4} . 10^{-14} . 250 . 4.10^{19}$

$= \pi . 250 . 10^{5}$

$\delta = \frac{1}{\sqrt{2\nu\pi}} = \frac{1}{\pi\sqrt{5.10^{7}}} = \frac{1}{7.\pi.10^{3}} = \frac{1}{2}.10^{-4}$

δ in this case 5 times as much

Krit. Temp. $\delta \neq \frac{1}{\sqrt[4]{\nu}} = 1\% = 3{\cdot}6$

To samo w bieżni gdzie 1 składnik optycznie aktywny

Czy interferencyjność zniszczona ?

Droga optyczna $(n)l$

$\Delta = (n)l\,\varepsilon.\delta < \frac{\lambda}{2}$

$l \neq \frac{\lambda_2}{\sqrt{2\nu\pi} . 10^{-4} . 3} = \frac{\quad}{3.10^{-5} . 3.10^{-4}\sqrt{2\pi\frac{\pi}{2^2}.4.10^{19} l}}$

$= \frac{10^{-1}}{\pi\sqrt{10^{20}}\, l} = \frac{10^{-1}}{10^{-8}.3.10^{10}\sqrt{l}} = \frac{10^{-3}}{3\sqrt{l}}$

$\sqrt{l} = 10^{-10}\frac{\sqrt{10^{20}}}{2\pi} = 10$

$\sqrt{\frac{8}{\pi^2\sigma^2 l N}} = \frac{2\sqrt{2}}{\pi\sigma\sqrt{lN}}$

$\frac{0{\cdot}00035 . 2\sqrt{2}\sqrt{l}}{3{\cdot}1 . 2{\cdot}9.10^{-8}\sqrt{4.10^{19}}}$

$= \frac{3{\cdot}5 . \cancel{2} . 10^{-5}}{3{\cdot}1 . \cancel{2{\cdot}9} . 3{\cdot}4} = \frac{1}{3} . 10^{5}\sqrt{l}$

$$= f(x,\omega) = f(1,1) + \Delta x\left(\frac{\partial f}{\partial x}\right)_{11} + \Delta\omega\left(\frac{\partial f}{\partial \omega}\right)_{11} + \frac{1}{2}\Delta x^2\left(\frac{\partial^2 f}{\partial x^2}\right)_{11} + \dots$$

$$\vartheta = \frac{\left(x + \frac{3}{\omega^2}\right)(3\omega - 1)}{8} = \frac{x}{8}(3\omega-1) + \frac{9}{8\omega} - \frac{3}{8\omega^2}$$

w bliskości punktu $\omega, x, \vartheta = 1$:

$$\left(\frac{\partial\vartheta}{\partial x}\right) = \frac{3\omega-1}{8}\Big| = \frac{1}{4} \qquad \frac{\partial\vartheta}{\partial\omega} = \frac{3x}{8} - \frac{9}{8\omega^2} + \frac{6}{8\omega^3}\Big| = 0$$

$$\left(\frac{\partial^2\vartheta}{\partial x^2}\right) = 0 \quad \dots\dots \quad \frac{\partial^2\vartheta}{\partial x\partial\omega} = \frac{3}{8} \qquad \frac{\partial^2\vartheta}{\partial\omega^2} = \frac{9}{4\omega^3} - \frac{9}{4\omega^4}\Big| = 0$$

$$\frac{\partial^3\vartheta}{\partial\omega^3} = \frac{9}{4}\left[-\frac{3}{\omega^4} + \frac{4}{\omega^5}\right]\Big| = \frac{9}{4}$$

$$\frac{\partial^n\vartheta}{\partial x^n} = 0 \qquad \frac{\partial^n\vartheta}{\partial x^n\partial\omega^{\dots}} = 0$$

$$\frac{\partial^4\vartheta}{\partial\omega^4} = \frac{9}{4}\left[\frac{12}{\omega^5} - \frac{20}{\omega^6}\right]\Big| = -18$$

$$\frac{\partial^n\vartheta}{\partial\omega^n}\Big| = \frac{9}{4}\left[\frac{n!}{2} - \frac{(n+1)!}{2.3}\right](-1)^{n-1}$$

$$= (-1)^{n+1}\frac{3}{8}\left[3n! - (n+1)!\right]$$

$$= (-1)^{n+1}\frac{3n!}{8}\left[3 - n - 1\right] = \frac{(-1)^{n+1}3(n-2)n!}{8}$$

$$\Delta\theta = \frac{\Delta x}{4} + \frac{1}{2}\left\{\frac{3}{4}\Delta x\Delta\omega\right\} + \frac{1}{2.3}\left\{\frac{9}{4}\Delta\omega^3\right\} - \frac{18}{2.3.4}\cdot\Delta\omega^4$$

$$= \frac{\Delta x}{4} + \frac{3}{8}\Delta x\Delta\omega + \frac{3}{8}\Delta\omega^3 - \frac{1}{4}\Delta\omega^4 + \dots$$

$$\Delta\theta \neq \frac{\Delta x}{4}\left(1 + \frac{3\Delta\omega}{2}\right) + \frac{3}{8}\Delta\omega^3$$

Istotn.: $$\Delta x = \frac{4\Delta\theta - \frac{3}{2}\Delta\omega^3}{1 + \frac{3\Delta\omega}{2}} = 4\Delta\theta - 6\Delta\theta\Delta\omega - \frac{3}{2}\Delta\omega^3$$

$$\oint \Delta x\, d\omega = 4\Delta\theta(\omega_1 - \omega_2) - 3\Delta\theta(\omega_1^2 - \omega_2^2) - \frac{3}{8}(\omega_1^4 - \omega_2^4)$$

$$= (\omega_1 - \omega_2)\left[4\Delta\theta - 6\Delta\theta\,\omega_1 - \frac{3}{2}\omega_1^3\right] = 0$$

$$4\Delta\theta - 3\Delta\theta(\omega_1+\omega_2) - \frac{3}{8}(\omega_1^3+\omega_1^2\omega_2+\omega_1\omega_2^2+\omega_2^3) - [\underbrace{4\Delta\theta + 6\Delta\theta\cdot\omega_1 - \tfrac{3}{2}\omega_1^3}_{=[4\Delta\theta - 3\Delta\theta(\omega_1+\omega_2) - \frac{3}{4}(\omega_1^3+\omega_2^3)]}] = 0$$

$$2(\omega_1^3+\omega_2^3) = (\omega_1^3+\omega_1^2\omega_2+\omega_1\omega_2^2+\omega_2^3)$$

$$\underbrace{\omega_1^3 - \omega_1^2\omega_2 - \omega_1\omega_2^2 + \omega_2^3}_{\omega_1^2(\omega_1-\omega_2) - \omega_2^2(\omega_1-\omega_2)} = 0$$

$$= (\omega_1-\omega_2)^2(\omega_1+\omega_2) = 0$$

$\underbrace{\Delta\omega_1 + \omega_2 = 0}$

Rathsas!

$\Delta\omega_1 = -\Delta\omega_2$ gerade Diameter! vertical

$$\left.\begin{aligned}\Delta\pi &= 4\Delta\theta - 6\Delta\theta\Delta\omega_1 - \tfrac{3}{2}\Delta\omega_1^3 \\ \Delta\pi &= 4\Delta\theta + 6\Delta\theta\Delta\omega_1 + \tfrac{3}{2}\Delta\omega_1^3\end{aligned}\right\}$$ unmöglich ausser:

Dampf-Spannungsgesetz $\left\{ \quad \frac{\Delta\pi}{\Delta\theta} = 4 \qquad \Delta\omega_1^2 = -\cancel{\ \ }\cdot 4\Delta\theta \right.$

$$\underline{\underline{\Delta\pi = 4\Delta\theta = -\Delta\omega^2}}$$

Boltzmann: p 53 (II)

$$c_p = \underbrace{\frac{3\pi(1+\beta)}{2}}_{c_v} + \underbrace{\frac{\pi}{1-\frac{2a(v-b)}{v(a+pv^2)}}}$$

$v = 3b$

$p = \frac{a}{27b^2}$

$$\frac{\pi}{1-\frac{2a\cdot 2b}{3b\ a(1+\frac{1}{3})}} = \frac{\pi}{1-\frac{4}{4}} = \infty \ !$$ vergl VdW I p 133

Adiabate: $v-b \sim \theta^{\frac{1}{k-1}}$

$$\left(\frac{v-b}{2b}\right)^{k-1} = \frac{\theta}{\theta_k} \qquad \vartheta_{w.} = \left(\frac{\omega-\frac{1}{3}}{\frac{2}{3}}\right)^{k-1} = \left(\frac{3\omega-1}{2}\right)^{k-1} = \left[1+\tfrac{3}{2}\Delta\omega\right]^{k-1}$$

$\Delta\theta = -(k-1)\frac{3}{2}\Delta\omega \quad \| \quad \Delta\pi = -(k-1)6.\Delta\omega \quad$ /Kr.

$$\left(\frac{\partial \pi}{\partial \theta}\right)_{\Delta\omega=cst} = \frac{4}{1+\frac{3}{2}\Delta\omega} \qquad \left(\frac{\partial \pi}{\partial \omega}\right)_{\Delta\theta=cst} = -\frac{9\,\Delta\omega^2}{8} \qquad \left(\frac{\partial \omega}{\partial \theta}\right)_{\Delta\pi} = +\frac{8}{9\Delta\omega^2}\;\frac{4}{1+\frac{3}{2}\Delta\omega} = \frac{32}{9\Delta\omega^2(1+\frac{3}{2}\Delta\omega)}$$

$$\delta\varphi = c_v\, d\theta + AT.\frac{4}{1+\frac{3}{2}\Delta\omega}\, d\omega$$

$$c_p = c_v + AT\,\frac{16\;.8}{(1+3\Delta\omega)\,9\,\Delta\omega^2}$$

$$= c_v + \frac{128.\,AT}{9\,\Delta\omega^2(1+3\Delta\omega)}$$

$$=$$

Andere Art der Messung f. ein Wechselgas

pro sec Anzahl der die Seitenwände durchfliegenden Molek. wenn n auf 1 cm²

$2(\Delta x \Delta y + \Delta y \Delta z + \Delta x \Delta z)\, n$ theils hinein theils heraus

$$\int_0^{\frac{\pi}{2}} \frac{2c\cos\varphi \cdot 2\pi \sin\varphi\, d\varphi}{2\pi} = \text{[illegible]}$$

$$p = \frac{Nmc^2}{3} = \frac{mc}{2} \cdot n$$

$$= 2c \frac{\sin^2}{2} = c$$

$$n = \frac{2}{3} N c$$

[illegible]

Drinnen sind normal enthalten: $N \Delta x \Delta y \Delta z$

Während t gesamt Zahl $2nt(\Delta x \Delta y + \ldots)$

Davon wahrscheinliche Abweichung vom Mittelwerth: ~~[illegible]~~ $\sqrt{2nt \ldots}$

Also kann man berechnen wie lange es durchschnittlich dauert bis die Abweichung verschwindet

Dazu aber vorausgesetzt dass die links eintretenden meistens rechts wieder austreten werden

also ist das nur gültig für $\begin{matrix}\Delta x \\ \Delta y \\ \Delta z\end{matrix} > \lambda$

Erst wenn die inneren Zusammenstöße unberücksichtigt, geht jedes von auswärts kommende Mol. hindurch

Bleibt also durchschnittlich während der Zeit $\frac{\Delta x}{3c}$ drinnen

Während dieses Zeitraumes sind überhaupt $\frac{2 \Delta y \Delta z\, n}{2} \frac{\Delta x}{3c}$ Mol. ~~[illegible]~~ angekommen, welche alle drinnen sind falls gleichmässig vertheilt

also sind gleichzeitig $= \frac{\Delta x \Delta y \Delta z \cdot n}{3c}$

Entropie = Log (Zustandswahrscheinlichkeit)

Zustandswahrsch. = e^{S}

Wahrsch. eines spec. Vol. $v \dots v + dv$, wenn das normale Vol. $\cdot v_0$ beträgt $W(v)\,dv$

$$\left.\begin{array}{l} W(v)\,dv = e^{S} \\ W(v_0)\,dv = e^{S_0} \end{array}\right\} \quad W(v) = W(v_0)\, e^{S - S_0}$$

Jaka różnica entropii w stanach v, v_0?

1). ~~Gaz~~ Gas sam się skurczył ~~bez pracy zewnętrznej~~ tak że sprężyna zewnętrzna napięciowa uciska się po wykonaniu pracy $p_0 dv$; bez ciepła z zewnątrz

$$\delta Q = \frac{\partial U}{\partial T} dT + \frac{\partial U}{\partial v} dv \cancel{+A}\, p_0\, dv = 0$$

zatem ~~podwyższenie~~ obniżenie temp. $$dT = -\frac{\frac{\partial U}{\partial v} \cancel{+A p_0}}{\frac{\partial U}{\partial T}} dv$$

To jest jego stan dla v

2). Przy ciągłym stanów równowagi ~~(ste)~~ sprowadzamy go napowrót do stanu $v_0\ T_0$:

$$\delta Q = \left(\frac{\partial U}{\partial T}\right) dT + \left(\frac{\partial U}{\partial v}\right) dv + A p\, dv = A(p - p_0)\, dv$$

$$S - S_0 = A \int \frac{(p - p_0)\, dv}{T}$$ ale tutaj ciśnienie p odpowiada normalnemu ciśnieniu gazu przy temp. $T \gtrless T_0$

np. tak że najpierw zmniejszamy p niezmiennego v tylko T aż dojdziemy do izotermy T_0 potem po tej izotermie wracamy do punktu pierwotnego

Wszystkie takie argumentacje nie mają sensu

bo termodynamicznie nie możemy wcale określić sposobu żadnego do obliczenia entropii S w takich stanach które nie odpowiadają normalnej równowadze!

Inna argumentacja:

$$S-S_0=\int \frac{\frac{\partial U}{\partial T}dT+\frac{\partial U}{\partial v}dv+Ap\,dv}{T}$$ niezależnie od kształtu drogi

. Stan v i v_0 posiadają tę samą temp. zatem $T=T_0$

$$S-S_0=\frac{1}{T_0}\left(U-U_0+A\underbrace{\int p\,dv}_{izot.}\right)$$

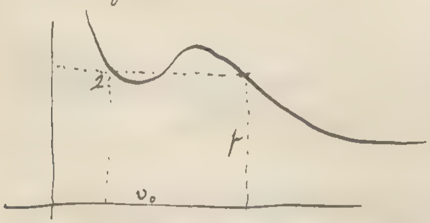
Dla gazu ideal. $S-S_0=AR\lg\frac{v}{v_0}=AR\lg(1+\delta)=-AR\delta$ fluktuacje!

Ale czy nie należy raczej wprowadzić:

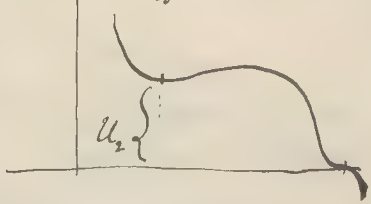
$$W=W_0\, e^{\frac{1}{T_0}\left[\underline{\underline{U-U_0}}+A\int(p-p_0)\,dv\right]}$$?

Przeciwnie, trzeba znaleźć prawdop. W z zasad kinetyki i na tej podstawie można określić co nazwać należy entropią w takich stanach anormalnych.

Wracając do dowodu ./.·/·/· /· ·/·



$$U=\int_{v_0}^{v} p\,dv-p_0(v-v_0)$$



gdy $U=0$ będziemy mieli całą podrównię $F_1=A\sqrt{\frac{\pi}{\alpha}}$ $\alpha=\left(\frac{\partial p}{\partial v}\right)_0\cdot\frac{N}{RT}\left(\frac{V}{v}\right)$

$F_2=A\sqrt{\frac{\pi}{\beta}}$ $\beta=\left(\frac{\partial p}{\partial v}\right)_2\cdot\frac{N}{RT}\left(\frac{V}{v}\right)$

więc prawdop. nie będą równe, ale gdy się dalej posuniemy (w razie jeśli było $F_1 > F_2$)

$$F_1=A\sqrt{\frac{\pi}{\alpha}};\quad F_2=e^{-U_2\frac{N}{RT}\frac{V}{v}}\sqrt{\frac{\pi}{\beta}}$$

więc z powodu olbrzymiości liczby N już nadzwyczajnie mały przyrost U_2 więc nadzwyczaj mała zmiana (p,v) wystarczy do zrównania $F_1=F_2$, a dalej do $F_1>F_2$ itd.

Jaka różnica energii wewnętrznej (potencjalnej) ~~...~~ powstaje jeśli gaz (gęstości ρ) pierwotnie jednostajnej gęstości tak zmienimy że element ω otrzyma ~~gęstość~~) $\rho(1+\delta)$
obj. ω, v

Ponieważ uważamy energię kinetyczną całkowitą niezmienioną więc

$= \delta U$ [różnica energii wewnętrznej termodynamicznej]

$$\delta U = \delta U_1 + \delta U_2 = \frac{\omega}{v}\int_{v_0}^{v_0(1-\delta)}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv + \frac{v-\omega}{v}\int_{v_0}^{v_0\left(1-\frac{\omega}{v_0-\omega}\delta\right)}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$

$$\omega(1+\delta) + (v_0-\omega)\,x = v_0$$

$$x = \frac{-\omega(1+\delta)+v_0}{v_0-\omega} = 1 - \frac{\omega}{v_0-\omega}\delta$$

$$= f(\delta) = f(0) + \delta\left(\frac{\partial f}{\partial \delta}\right)_0 + \frac{\delta^2}{2}\left(\frac{\partial^2 f}{\partial \delta^2}\right)_0$$

$$f(0) = 0$$

$$\frac{\partial f}{\partial \delta} = \frac{\omega}{v}$$

$$\frac{v_0}{v} = 1+\delta$$

$$(1+\delta)^\nu e^{\nu\left(\frac{1}{1+\delta}-1\right)} = \left[(1+\delta)\,e^{-\frac{\delta}{1+\delta}}\right]^\nu = \left[(1+\delta)\left[1-\frac{\delta}{1+\delta}+\frac{1}{2}\left(\frac{\delta}{1+\delta}\right)^2\ldots\right]\right]^\nu$$

$$= \left[1+\delta-\delta+\frac{1}{2}\frac{\delta^2}{1+\delta}\ldots\right]^\nu$$

$$= \left[1+\frac{\delta^2}{2}\right]^\nu = e^{\frac{\nu\delta^2}{2}}$$

$$(1+x)^{\frac{\alpha}{x}} = e^\alpha$$

$$(1+x)^y = e^{xy}$$

$$\frac{\nu}{n} = e^{\lg\frac{\nu}{n}} = e^{-\lg(1+\delta)} = e^{-\delta+\frac{\delta^2}{2}-\frac{\delta^3}{3}}$$

$$e^{1-\frac{\nu}{n}} = e^{+\frac{\delta}{1+\delta}} = e^{+\delta-\delta^2--}$$

I objętości dana, wiele różne [illegible]
II ilość n dana, wiele różne objętości

Dla gazu idealnego

Sposób I: $\log W = n \log \frac{\nu}{n} + (n-\nu) \cancel{\ }\!- \frac{1}{2}\log 2\pi n + \log dn$

II: $dW = \underline{A}\, dv \cdot e^{+hq\left[\int_{v_0}^{v} p_v\, dv - p_0(v-v_0)\right]}$ $\qquad \rho = \frac{V_0}{v_0}$

$$dW = \underline{A}\, e^{-h\frac{m(\xi^2+\eta^2+\zeta^2)}{2}} d\xi\, d\eta\, d\zeta \quad = \frac{1}{n}\left(\frac{mh}{2\pi}\right)^{3/2} e^{-h\frac{m}{2}(\xi^2+\eta^2+\zeta^2)} d\xi \ldots$$

$$1 = \underline{A}\left(\sqrt{\frac{2\pi}{mh}}\right)^3$$

$$\int \xi^2 e^{-\frac{hm\xi^2}{2}} d\xi = \frac{1}{hm}\int e^{-h\ldots}$$

$$p = n m \overline{\xi^2}$$

$$= \frac{nm}{hm} = \frac{\rho}{hm} \qquad \frac{p}{\rho} = RT = \frac{1}{hm}$$

$$\overline{c^2} = \frac{\int 4\pi c^4\, dc\, e^{-\frac{hmc^2}{2}}}{\int 4\pi c^2\, dc\, e^{-\ldots}} = \frac{3 \cdot 2}{2\, hm} = \frac{3}{hm}$$

$$p = \frac{n m \overline{c^2}}{3} = \frac{\rho\, 3}{3 hm} \qquad \boxed{hm = \frac{1}{RT}} \qquad h = \frac{1}{mRT} = \frac{n}{\rho_0\, RT}$$

$$hq = \frac{V_0\, n}{v_0 \rho_0\, RT} = \frac{V_0\, n}{RT} = \frac{N}{RT}$$

$$\int_{v_0}^{v} p\, dv = RT\int \frac{dv}{v} = RT \log\frac{v}{v_0} = p_0 v_0 \log\frac{v}{v_0}$$

$N = (\nu)$

$\frac{v_0}{v}(\nu) = (n)$

$$dW = A\, dv\, e^{+\frac{N}{RT}\left[RT\log\frac{v}{v_0} - RT\left(\frac{v}{v_0}-1\right)\right]}$$

$v = \frac{\nu v_0}{n}$

$$= A\, dv\, e^{+N\left[\log\frac{v}{v_0} + 1 - \frac{v}{v_0}\right]} \quad = \cancel{A\, dv \cdot \left(\frac{v}{v_0}\right)^N e^{\ldots}}$$

$$= A\, dv \left(\frac{v}{v_0}\right)^N e^{N\left(1-\frac{v}{v_0}\right)} = \underline{A}\, dv \left(\frac{\nu}{n}\right)^{\nu} e^{\nu\left(1-\frac{\nu}{n}\right)} \cdot \sqrt{\frac{\nu}{2\pi}}\, \frac{\nu}{n^2}\, dn \cdot \left[\frac{\nu}{n} e^{1-\frac{\nu}{n}}\right]^{\nu}$$

I pisane z użyciem symbolów II:

$$\lg W = \frac{v_0}{v} N \lg \frac{v}{v_0} + \frac{v_0}{v} N \left(1 - \frac{v}{v_0}\right) - \frac{1}{2} \lg 2\pi \frac{v_0 N}{v} + \lg v_0 N$$

$$= N \left[\frac{v_0}{v} - 1 + \frac{v_0}{v} \lg \frac{v}{v_0} \right] - \frac{1}{2} \lg 2\pi \frac{v_0 N}{v}$$

$$dW = - \frac{e^{N\left[\frac{v_0}{v} - 1 + \frac{v_0}{v} \lg \frac{v}{v_0}\right]}}{\sqrt{2\pi \frac{v_0 N}{v}}} \cdot \frac{v_0 N}{v^2}\, dv$$

$$= - \frac{e^{-N\frac{v_0}{v}\left[1 - \frac{v}{v_0} - \lg \frac{v}{v_0}\right]}}{\sqrt{\cdots}} \cdots = -\left(\frac{v}{v_0}\right)^{N\frac{v_0}{v}} \frac{e^{-N\frac{v_0}{v}\left(1 - \frac{v}{v_0}\right)}}{\sqrt{\cdots}} \cdots dv$$

$$\frac{N^N}{n^n\,(N-n)^{N-n}}\left(\frac{v}{V}\right)^n\left(\frac{V-v}{V}\right)^{N-n}\sqrt{\frac{N}{n(N-n)}}$$

$$\int\left(\frac{\nu}{n}\right)^{\nu} e^{n-\nu}\,dn = e^{-\nu}\cdot\nu^{\nu}\int.$$

$$= \cancel{\frac{\nu^{\nu}}{n^n(\nu-n)^{\nu-n}}(\alpha)^n(1-\alpha)^{\nu-n} = \left(\frac{\nu}{\nu-n}\right)^{\nu}\left(\frac{\nu-n}{n}\right)^n\alpha^n(1-\alpha)^{\nu-n}}$$

$$\nu = \frac{Nv}{V}$$

$$= \left[\frac{N}{N-n}\ \frac{V-v}{V}\right]^N\left[\frac{\cancel{V}}{V-v}\cdot\frac{v}{\cancel{V}}\,\frac{N-n}{n}\right]^n\sqrt{\frac{N}{n(N-n)}}$$

$$\frac{\nu!}{n!\,\nu-n!} = \frac{\nu^{\nu}}{n^n(\nu-n)^{\nu-n}}\,\frac{e^{n-\nu}}{\sqrt{2\pi}}\sqrt{\frac{\nu}{n(\nu-n)}}$$

$$= \frac{\nu^{n+\nu-n}}{n^n(\nu-n)^{\nu-n}}\,e\cdots\cdots = \left(\frac{\nu}{n}\right)^n\cdot\left(\frac{\nu}{\nu-n}\right)^{\nu-n}e\cdots$$

$$= \left(\frac{\nu}{n}\right)^n(\cancel{\ })$$

$$= \left[\frac{N-\nu}{N-n}\right]^N\left[\frac{\nu-n\frac{v}{V}}{(1-\frac{v}{V})n}\right]^n$$

$$= \left(\frac{1-\frac{\nu}{N}}{1-\frac{n}{N}}\right)^N\left(\frac{\nu}{n}\right)^n\left[\frac{1-\frac{n}{\nu}\frac{v}{V}}{1-\frac{v}{V}}\right]^n$$

stimmt

$$= \frac{e^{-\nu}}{e^{-n}}\left(\frac{\nu}{n}\right)^n$$

$$\frac{vN}{v_0} = x \qquad \frac{v}{v_0} = \frac{x}{N} \qquad dv = dx\,\frac{v_0}{N}$$

$$A\,e^N\int_0^{\infty} e^{-\frac{vN}{v_0}}\left(\frac{v}{v_0}\right)^N dv = A\,e^N\int_0^{\infty} e^{-x}\left(\frac{x}{N}\right)^N\frac{v_0}{N}\,dx = A\,\frac{e^N v_0}{N^{N+1}}\int_0^{\infty} e^{-x}x^N\,dx$$

$$= A\,\frac{e^N v_0}{N^{N+1}}\,\underbrace{\Gamma(N+1)}_{=\sqrt{2N\pi}\left(\frac{N}{e}\right)^N}$$

$$= \cancel{\frac{A v_0\,e^N\sqrt{2\pi(N+1)}\,(N+1)^{N+1}}{N^{N+1}\,e^{N+1}}}$$

$$= \cancel{\frac{A v_0}{e}\sqrt{2\pi(N+1)}\left(1+\frac{1}{N}\right)^{N+1}}$$

$$= \frac{A\,e^N v_0}{N^{N+1}}\,\frac{\sqrt{2N\pi}\;N^N}{e^N} = \frac{A v_0\sqrt{2N\pi}}{N}$$

$$= A v_0\sqrt{\frac{2\pi}{N}} = 1$$

$$A = \sqrt{\frac{N}{2\pi}}\,\frac{1}{v_0}$$

$$dW = A\, dW_0\, e^{-hU}$$

dW_0 = prawdp. gdyby nie było energii ~~...~~ potencyalnej U

więc dla gazu idealnego

U = energia potencyalna danego układu

$$\underbrace{\underline{n} \text{ w } \underline{v},\ \underline{N-n} \text{ w } \underline{V-v}}$$

$$U = U_0 + \delta U$$

$$A e^{-hU_0} = \underline{A'}$$

$$\int_{n=0}^{\infty} A'\, dW_0\, e^{-h\delta U} = 1$$

$$\delta U = n m \int_{v_0}^{\frac{v}{n}v_0} \quad + \underbrace{(N-n) m \int_{v_0}^{v_0 \cdot \frac{N-v}{N-n}} \left[T \frac{\partial p}{\partial T} - p\right] dv}$$

ale tutaj także implicite

[?] pewnych założeń?

pomiar

$$(N-n) m \int_{\left(1+\frac{v-n}{N}\right)v_0}^{v_0} \cdots\cdots$$

$$\frac{v}{n} v_0 = v$$

$$= -(N-n) m \left[T \left(\frac{\partial p}{\partial T}\right) - p\right]_0 \cdot v_0 \frac{v-n}{N} \neq - v_0 (v-n) m \left[T \frac{\partial p}{\partial T} - p\right]_0$$

$$\delta U = n m \left[\int_{v_0}^{v} \left[T \frac{\partial p}{\partial T} - p\right] dv + (v_0 - v) \left[T \frac{\partial p}{\partial T} - p\right]_0\right]$$

$$dW = A \left(\frac{v}{n}\right)^n \frac{e^{n-v}}{\sqrt{2n\pi}}\, dn \cdot e^{-\frac{n}{p_0 v_0}\left[\int_{v_0}^{v} \left(T \frac{\partial p}{\partial T} - p\right) dv + (v_0 - v) \left[T \frac{\partial p}{\partial T} - p\right]_0\right]}$$

$$\int_{v_0}^{v}\left(T\frac{\partial p}{\partial T}-p\right)dv = F(v)-F(v_0)$$

$$= F[v_0+(v-v_0)]-F(v_0)$$

$$=(v-v_0)\left.\frac{\partial F}{\partial v}\right|_0+\frac{(v-v_0)^2}{2}\left.\frac{\partial^2 F}{\partial v^2}\right|_0+\cdots$$

$$=(v-v_0)\left[T\frac{\partial p}{\partial T}-p\right]_0+\frac{(v-v_0)^2}{2}\frac{\partial}{\partial v}\left[T\frac{\partial p}{\partial T}-p\right]$$

$$p+\frac{a}{v^2}=\frac{RT}{v-b}$$

$$\underbrace{\int_{v_0}^{v}\left(T\frac{\partial p}{\partial T}-p\right)dv=\frac{a}{v_0}-\frac{a}{v} \qquad (v_0-v)\left[T\frac{\partial p}{\partial T}-p\right]_0=(v_0-v)\frac{a}{v_0^2}}$$

$$\frac{a}{v_0}\left[1-\frac{v_0}{v}+\frac{v_0-v}{v_0}\right]=\frac{a}{v_0}\left[2-\frac{v_0}{v}-\frac{v}{v_0}\right]=-\frac{a\,(v_0-v)^2}{v_0^2\,v}$$

To nie miałoby zatem żadnych szczególnych własności w punkcie krytycznym!

Dla małych v_0-v : $v_0=v(1+\delta)$:

$$dW = A\,e^{-\frac{v\delta^2}{2}+\frac{n}{p_0v_0}\frac{a\,v\,\delta^2}{v_0^2}} \neq A\,e^{-\frac{v\delta^2}{2}\left[1-\frac{2a}{p_0v_0^2}\right]}$$

Wykładnik $=0$ jeśli $p=\frac{2a}{v^2}$ tj.: $\frac{3a}{v^2}=\frac{RT}{v-b}$

podczas gdy w punkcie krytycznym: $\left.\begin{array}{l} v_k=3b \\ p_k=\frac{a}{27b^2} \end{array}\right\}=\frac{a}{3v_k^2}$

Powinno być:

$$\underset{(148)}{-P} = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}\underbrace{\frac{2\varphi-2\varphi'-\varphi''}{2\varphi+\varphi'}}_{M} + \delta\left\{\frac{4\varphi'}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}\underbrace{\frac{6\varphi'^2+6\varphi\varphi''+2\varphi\varphi''+\varphi''^2}{(2\varphi+\varphi')^2}}_{N}\right\} + \Pi(1-6\delta)$$

$$M = 1 - \frac{3\varphi'+\varphi''}{2\varphi+\varphi'}$$

$6\cancel{\varphi\varphi''} \cancel{+3\varphi'\varphi''} - \cancel{6\varphi'^2} - 2\varphi'\varphi''$ $\cancel{-3\varphi\varphi'}$ $-\varphi'^2$

$$\frac{\partial M}{\partial l} = -\frac{(2\varphi+\varphi')3\varphi'' - (3\varphi'+\varphi'')(2\varphi'+\varphi'')}{(2\varphi+\varphi')^2}$$

$$\frac{4\varphi}{\sqrt{2}} - \frac{M\,rT}{2\sqrt{2}} = -\Pi\, l^2 \qquad M > 0$$

$$\cancel{\frac{4\varphi'}{\sqrt{2}} - \frac{M'}{2\sqrt{2}h} = \frac{\partial\Pi}{\partial l} = -6\Pi}$$

$$\cancel{\frac{4\varphi'}{\partial l} - \frac{Mr}{}} \quad \left(\frac{4\varphi'}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}M'\right)\frac{\partial l}{\partial T} - \frac{Mr}{2\sqrt{2}} = \underbrace{+\cancel{6}\Pi\frac{\partial l}{\partial T}l}_{= -\frac{\partial\Pi}{\partial T}\cdot l^2 - 2l\Pi\frac{\partial l}{\partial T}}$$

$$\frac{\partial l}{\partial T} = \frac{\frac{Mr}{2\sqrt{2}}}{\frac{4\varphi'}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}M' - 6\Pi} \qquad \Pi = \frac{1}{l^2}\left[-\frac{4\varphi}{\sqrt{2}} + \frac{MrT}{2\sqrt{2}}\right]$$

$$\overline{U} = n\left\{6\overset{?}{\Phi} + \frac{3}{2}rT\right\} \qquad n m = \rho \qquad n\,l^3 = \sqrt{2} \qquad n\,l^2\cdot\frac{l}{\sqrt{2}} = 1$$

$$c\rho = \frac{\partial U}{\partial T} = n\left[6\varphi\cdot\frac{\partial l}{\partial T} + \frac{3r}{m}\right] + 3\frac{\Pi}{l}\frac{\partial l}{\partial T} = n\left\{\frac{\partial l}{\partial T}\left[6\varphi + \frac{3}{nl^3}\left(-\frac{4\varphi}{\sqrt{2}} + \frac{MrT}{2\sqrt{2}}\right)\right] + \frac{3r}{m}\right\}$$

$$\cancel{c = \frac{n}{m}\left[6\varphi\frac{\partial l}{\partial T} + \frac{3r}{2}\right] + \frac{3\Pi}{\rho}\frac{\partial l}{\partial T}} = n\left\{\frac{\partial l}{\partial T}\left[\cancel{\ \ }+ \frac{3MrT}{4}\right] + \frac{3r}{m}\right\}$$

$$\frac{3}{2}rT + 6\delta\varphi' + 3\delta^2\varphi''$$

Superpozycja Próba

Robię po kolei rozszerzenie $(1+\delta)$ w kierunku X Y Z :

$$-\mathfrak{P} = \frac{4\varphi}{\sqrt{2}} \div \frac{1}{2\sqrt{2}h} \frac{2\varphi - 2\varphi' l - \varphi' l^2}{2\varphi + \varphi' l}$$

$$+\delta\left\{ \frac{4\varphi' l}{\sqrt{2}} + \frac{1}{2\sqrt{2}h} \frac{-32\varphi^2 - 32\varphi\varphi' + 64\varphi^2 - 64\varphi\varphi'' + 8\varphi''^2 + 16\varphi'\varphi''}{8(2\varphi+\varphi')^2} \right\}$$

$$= \frac{-4\varphi^2 - 4\varphi\varphi' + 8\varphi'^2 - 8\varphi\varphi'' + \varphi'^2 + 2\varphi'\varphi''}{(2\varphi+\varphi')^2}$$

Nie zgadza się!

$$(2\varphi+\varphi' l)(2\varphi' - 2\varphi'') - (2\varphi - 2\varphi' - \varphi')(2\varphi' + \varphi'') =$$

$$= 4\varphi\varphi' + 2\varphi'^2 - 4\varphi\varphi'' - 2\varphi'\varphi''$$
$$-4\varphi\varphi' + 4\varphi'^2 - 2\varphi\varphi'' + 2\varphi'\varphi'' + 2\varphi'\varphi'' + \varphi''^2$$

$$= \frac{6\varphi'^2 - 6\varphi\varphi'' + 2\varphi'\varphi'' + \varphi''^2}{4\varphi^2 + 4\varphi\varphi' + \varphi'^2}$$

$$-l^2 X = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,h}\,\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l} + \qquad + \Pi(1-2\delta)$$

$$+\,\delta\left\{\frac{2\varphi}{\sqrt{2}} + \frac{2\varphi' l}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,h}\,\frac{3\varphi^2 - 30\varphi\varphi' + 27\varphi'^2 - 38\varphi\varphi'' + 3\varphi''^2 + 2\varphi'\varphi''}{8\,(2\varphi + \varphi' l)^2}\right\}$$

$$-l^2(1+\delta)\,Y = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,h}\,\frac{2\varphi \cdot 2\varphi' l \cdot \varphi'' l^2}{2\varphi + \varphi' l} + \qquad + \Pi(1-2\delta)$$

$$+\,\delta\left\{-\frac{\varphi}{\sqrt{2}} + \frac{\varphi' l}{\sqrt{2}} + \frac{1}{2\sqrt{2}\,h}\,\frac{35\varphi^2 + 2\varphi\varphi' - 37\varphi'^2 + 26\varphi\varphi'' - 5\varphi''^2 - 14\varphi'\varphi''}{16\,(2\varphi + \varphi')^2}\right\}$$

$$\frac{\cancel{\frac{4\varphi}{\sqrt{2}}}\,(u^2+v^2+w^2)\, e^{-h(u^2+v^2+w^2)\frac{m}{2}}}{\ldots} = \frac{3}{hm} = \bar{c}^2 \qquad \frac{p}{\rho} = \frac{\bar{c}^2}{3} = \frac{1}{hm} = RT$$

$$\frac{1}{h} = RTm = rT$$

$$\alpha = \frac{1}{l}\frac{\partial l}{\partial T}$$

$$-\frac{4\varphi}{\sqrt{2}} + \frac{rT}{2\sqrt{2}}\,\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l} = +\Pi$$

$M \lessgtr 0$

$$\left[\frac{4\varphi'}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}\frac{\partial M}{\partial l} - 6\Pi\right]\alpha l - \frac{r}{2\sqrt{2}}M = 0$$

$$\alpha l = \frac{\frac{r\,M}{2\sqrt{2}}}{\frac{4(\varphi' + 6\varphi)}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}\left(\frac{\partial M}{\partial l} + 6M\right)}$$

~~zależy od~~ ~~znaku~~ ~~temperat.~~

w gorszem przybliżeniu, dla ~~$\Pi = 0$~~

$$\Pi = -\frac{4\varphi}{\sqrt{2}} - \frac{rT}{\sqrt{2}}$$

$$\cancel{\alpha l = \frac{rM}{2(\varphi' + 6\varphi)}}$$

$$0 = -\frac{4\varphi'}{\sqrt{2}}\cdot\alpha l - \frac{r}{\sqrt{2}} \qquad \alpha l = -\frac{4\varphi'}{r}$$

nonsens!

$$\frac{1}{a-b\delta} = \frac{1}{a\left(1-\frac{b}{a}\delta\right)} = \frac{1}{a} + \frac{b\delta}{a^2}$$

$$\int \left[\frac{3(x^2+y^2)\delta}{4\sqrt{2}}(\varphi-\varphi' l) + \frac{z^2\delta}{2\sqrt{2}}\left[3(\varphi-\varphi') + \varphi''\right] \right] e^{-h\left[(x^2+y^2+z^2)\, 2(2\varphi+\varphi') - x^2\delta[3(\varphi-\varphi')-\varphi''] - \frac{y^2+z^2}{2}\delta[\varphi-\varphi'-\varphi'']\right]}$$

$$- \frac{x^2+y^2}{2\sqrt{2}}(\varphi-\varphi'-\varphi'') + \frac{y^2}{\sqrt{2}}\left[3\varphi - 3\varphi' - \varphi''\right]$$

$$= \frac{\delta}{2h} \left\{ \frac{3(\varphi-\varphi' l)}{4\sqrt{2}} \left\{ \frac{1}{\left[4\varphi+2\varphi' - \delta(3\varphi-3\varphi'-\varphi'')\right]} + \frac{1}{\left[4\varphi+2\varphi' - \frac{\delta}{2}(\varphi-\varphi'-\varphi'')\right]} \right\} \right.$$

$$\left. + \frac{3\varphi-3\varphi'+\varphi''}{2\sqrt{2}} \frac{1}{4\varphi+2\varphi'-\frac{\delta}{2}(\varphi-\varphi'-\varphi'')} \right\} -$$

$$- \frac{1}{2\sqrt{2}}\frac{1}{2h}\left\{ \frac{(\varphi-\varphi'-\varphi'')}{4\varphi+2\varphi'-\delta(3\varphi-3\varphi'-\varphi'')} + \frac{\varphi-\varphi'-\varphi''+6\varphi-6\varphi'-2\varphi''}{4\varphi+2\varphi'-\frac{\delta}{2}(\varphi-\varphi'-\varphi'')} \right\}$$

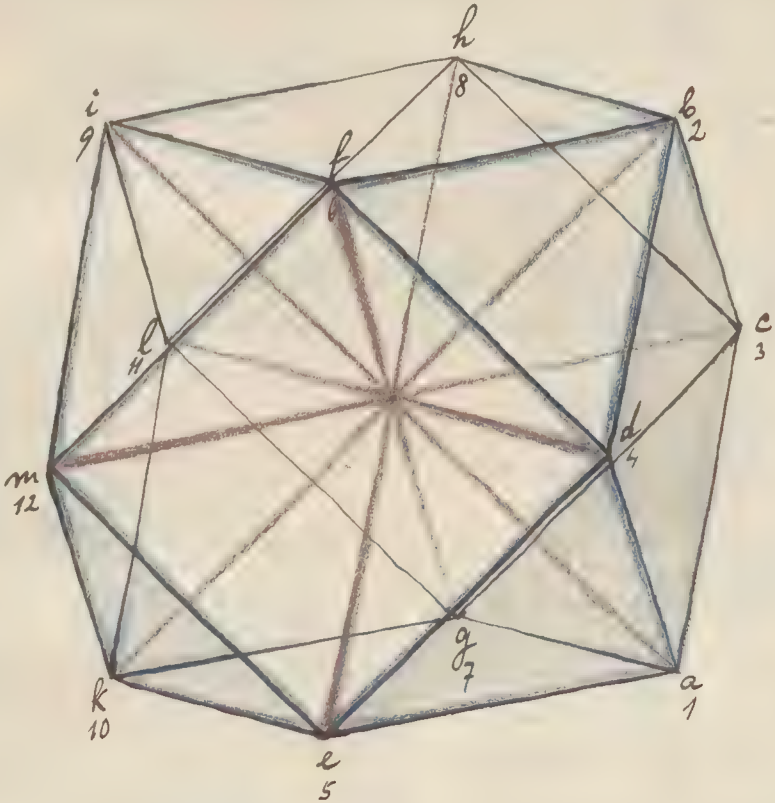
$$= -\frac{1}{4\sqrt{2}\,h} \frac{8\left(\varphi-\varphi'-\frac{\varphi''}{2}\right)}{4\varphi+2\varphi'}$$

$$+ \frac{\delta}{4\sqrt{2}\,h}\left\{ \frac{3(\varphi-\varphi') + 3\varphi - 3\varphi' + \varphi''}{4\varphi+2\varphi'} - \frac{(\varphi-\varphi'-\varphi'')(3\varphi-3\varphi'-\varphi'') + (7\varphi-7\varphi'-3\varphi'')\left(\varphi-\varphi'-\frac{\varphi''}{2}\right)}{(4\varphi+2\varphi')^2} \right.$$

$$\frac{6\varphi-6\varphi'+\varphi''}{4\varphi+2\varphi'} - \frac{(\varphi-\varphi'-\varphi'')(13\varphi-13\varphi'-5\varphi'')}{2(4\varphi+2\varphi')^2}$$

$$= \left\{ \begin{array}{l} 48\varphi^2 - 48\varphi\varphi' + 84''\varphi \\ \quad + 24\varphi\varphi' \quad - 24\varphi'^2 + 4\varphi'\varphi'' \\ -13\varphi^2 + 13\varphi\varphi' + 5\varphi\varphi'' \quad - 13\varphi'\varphi'' \\ \quad 13\varphi\varphi' + 13\varphi\varphi'' - 13\varphi'^2 - 5\varphi'\varphi'' + 5\varphi''^2 \end{array} \right.$$

$$34 + \frac{\delta}{4\sqrt{2}\,h} \frac{35\varphi^2 + 2\varphi\varphi' + 26\varphi\varphi'' - 37\varphi'^2 - 14\varphi'\varphi'' - 5\varphi''^2}{8(2\varphi+\varphi')^2}$$


h 8
i 9
f 6
b 2
c 3
l 11
d 4
m 12
g 7
k 10
a 1
e 5

$\hat{in}$: $E = \underline{1200}.098.10^9$ (CGS)

$\mp 1{,}2.10^6$ atm.

$l^2 E \mp (\varphi + \varphi' l)\sqrt{2}$

$$-\frac{\partial U}{\partial r} = -4(2\varphi+\varphi')\frac{r}{l} = +m\frac{d^2r}{dt^2} \qquad \alpha = 2\sqrt{(2\varphi+\varphi')\frac{1}{ml}} \mp \sqrt{\frac{2l^2E}{ml}} = \sqrt{\frac{2\rho E}{l}}$$

$$r = r_0 \sin\alpha t$$

jeśli $\varphi' l \gg \varphi$

$$\frac{dr}{dt} = \alpha r_0 \cos\alpha t$$

$$\frac{2}{\pi}\int\left(\frac{dr}{dt}\right)^2 dt = \alpha^2 r_0^2 \frac{2}{\pi}\int_0^{\alpha t = \frac{\pi}{2}} \cos^2\alpha t\, dt = \frac{\alpha^2 r_0^2}{2} = \bar{c}^2$$

$$r_0 = \sqrt{\frac{2\bar{c}^2 ml}{4(2\varphi+\varphi')}} \mp \sqrt{\frac{2\bar{c}\, ml}{2\sqrt{2}\, l^4 E}}$$

$\bar{c} = 3.10^4$

$\rho = 10$

$E = 1{,}2.10^{12}$

$\cancel{\frac{r_0}{l}}$

$$\frac{r_0}{l} \mp \sqrt{\frac{\bar{c}^2 m}{\sqrt{2}\, E\, l^3}}$$

$N l^3 = 1$

$N m = \rho$

$\frac{l^3}{m} = \frac{1}{\rho}$

$$\frac{r_0}{l} = \sqrt{\frac{9.10^8.10}{\sqrt{2}.1{,}2.10^{12}}} \mp \sqrt{\frac{0{,}9.10^{10}}{1{,}8.10^{12}}} = \sqrt{\frac{1}{200}} = \frac{1}{14} = 7\%$$

$$\mp \bar{c}\sqrt{\frac{\rho}{\sqrt{2}\, E}}$$

$$\sqrt{\frac{\alpha T\ \text{atom volum}}{\sqrt{2}\, E}}$$

Przeciętne oddalenie się cząstek z pozycji równowagi $r_0 = \frac{l}{14}$

Wówczas coś tu wykresu nie mniejszy (atom volum. $\times E$)

Dla $E = 0$ (punkt topliwości) $r_0 = \infty$ zupełne oddalenie

$$T = \frac{2\pi}{\alpha} \qquad n = \frac{\alpha}{2\pi} = \frac{\sqrt{2\rho E}}{2l\pi} = \frac{\sqrt{20.1{,}2.10^{12}}}{2.3{,}1.10^{-8}} = \frac{5}{6{,}3}.10^{14} \mp 10^{14}$$

$$\lambda = \frac{3.10^{10}}{n} = \frac{3.10^{10}}{10^{14}} = 3.10^{-4} = \underline{\underline{3\mu\ !!!}}$$

ciągnienie

$$X = \frac{4\varphi}{\sqrt2}\left(1+\frac{\delta}{2}\right) + 2\frac{\varphi l \delta}{\sqrt2} - \frac{1}{2\sqrt2 k}(M + N\delta) + \Pi(1-2\delta)$$

$$Y = \frac{4\varphi}{\sqrt2}\left(1-\frac{5\delta}{4}\right) + \frac{\varphi' l \delta}{\sqrt2} - \frac{1}{2\sqrt2 k}\left(M + (N'-M)\delta\right) + \Pi(1-3\delta)$$

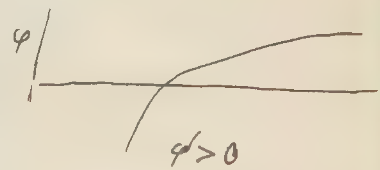
$$0 = \frac{4\varphi}{\sqrt2} - \frac{M}{2\sqrt2 k} + \Pi \qquad \Pi = -\frac{4\varphi}{\sqrt2} + \frac{M}{2\sqrt2 k}$$

$$+c_{11} = \frac{2\varphi}{\sqrt2} + \frac{2\varphi' l}{\sqrt2} - \frac{N}{2\sqrt2 k} - 2\Pi$$

$$+c_{11} = \frac{10\varphi}{\sqrt2} + \frac{2\varphi' l}{\sqrt2} - \frac{(N+2M)}{2\sqrt2 k}$$

$$+c_{21} = -\frac{5\varphi}{\sqrt2} + \frac{\varphi' l}{\sqrt2} - \frac{(N'-M)}{2\sqrt2 k} - 3\Pi$$

$$+c_{21} = \frac{7\varphi}{\sqrt2} + \frac{\varphi l}{\sqrt2} - \frac{(N'+2M)}{2\sqrt2 . k}$$



$\varphi' > 0$

$\varphi'' < 0$

$$c_{44} = \frac{3\varphi + \varphi'}{\sqrt2} - \frac{(\varphi - \varphi' - \varphi'')^2}{4\sqrt2(2\varphi+\varphi')} - \frac{4\varphi - 4\varphi' - 10\varphi''}{16\sqrt2(2\varphi+\varphi')k}$$

$$\frac{(1-\mu)}{\mu} = \begin{array}{l} 30\varphi + 6\varphi' l \\ +14 \quad +2 \\ +12 \quad +4 \\ \hline \end{array}$$

$$\frac{56\varphi + 12\varphi' l}{\sqrt2} - \frac{(\varphi-\varphi'-\varphi'')^2}{\sqrt2(2\varphi+\varphi')}$$

$$\begin{array}{l} 10 + 2 \\ 28 \quad 4 \\ -6 \quad -2 \\ \hline \end{array}$$

$$\frac{32\varphi + 4\varphi' l}{\sqrt2} + \frac{(\varphi-\varphi'-\varphi'')^2}{2\sqrt2(2\varphi+\varphi')}$$

---

czem większe φ (ujemne) tem mniejsze μ

$$\mu = \frac{1}{1 + \dfrac{56\varphi + 12\varphi' l}{32\varphi + 4\varphi' l}} = \frac{32\varphi + 4\varphi' l}{88\varphi + 16\varphi' l} = \frac{\varphi' l + 8\varphi}{4\varphi' l + 22\varphi}$$

$\cancel{\Phi} =$

$$\cancel{12\,\Phi(l) + \varphi l\left[4\delta + \tfrac{\delta}{4}(x^2+y^2+z^2) - 2\,3x^2\delta - \tfrac{5x^2}{2}\delta\right.}$$

$$12\,\Phi(l) + \varphi\, l\left\{4\delta + [(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]\left(4-\frac{\delta}{2}\right) - \frac{5(x-\xi)^2\delta}{2}\right\}$$

$$+\varphi' l^2\left\{\quad [\cdot\ \cdot\ \cdot\ -\ \ -\ -\ -]\left(2+\frac{\delta}{2}\right) + \frac{5(x-\xi)^2\delta}{2}\right\}$$

$$+\varphi'' l^3\left\{\quad [\cdot\ -\quad]\ \frac{\delta}{2} + \frac{(x-\xi)^2\delta}{2}\right\}$$

$$U = 24\,\Phi(l) + 8\varphi l\delta + \left[\varphi\, l\left(4-\frac{\delta}{2}\right) + \varphi' l^2\left(2+\frac{\delta}{2}\right)\right](x^2+y^2+z^2+\xi^2+\eta^2+\zeta^2)$$

$$+\varphi''\frac{\delta}{2}$$

$$-\left[\frac{5\delta}{2}(\varphi l - \varphi' l^2) + \frac{\delta}{2}\varphi'' l^3\right](x^2+\xi^2)$$

$$-\Phi - \varphi\left\{\frac{\delta}{2} - \frac{x\xi}{l^2}\left(1+\frac{\delta}{2}\right) + \frac{y\eta}{l^2}\left(1-\frac{\delta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\delta}{2}\right) + \frac{xy}{2}\left(1-\frac{\delta}{2}\right)\right.$$

$$-\Phi - \varphi\left\{\frac{\delta}{2} + \frac{\xi}{l^2}\left(1+\frac{\delta}{2}\right) - \frac{\eta}{l^2}\left(1+\frac{\delta}{2}\right) + \frac{\xi^2+\eta^2+\zeta^2}{2}\left(1-\frac{\delta}{2}\right) - \frac{\xi^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{\eta^2}{4}\left(1-\frac{3\delta}{2}\right) + \frac{\xi\eta}{2}\left(1-\frac{\delta}{2}\right)\right.$$

$$+\Phi + \varphi\left\{\frac{\delta}{2} - \frac{x-\xi}{l^2}\left(1+\frac{\delta}{2}\right) + (y-\eta)\ldots\right\} + \frac{(x-\xi)^2+\ldots}{2} - \frac{(x-\xi)^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{(y-\eta)^2}{4}\left(1-\frac{3\delta}{2}\right) + \frac{(x-\xi)(y-\eta)}{2}\left(1-\frac{\delta}{2}\right)$$

$$-\Phi - \varphi\left\{\frac{\delta}{2} + (x\xi+y\eta+z\zeta)\left(1-\frac{\delta}{2}\right) + \frac{x\xi}{2}\left(1+\frac{\delta}{2}\right) + \frac{y\eta}{2}\left(1-\frac{3\delta}{2}\right) + \frac{\xi y+x\eta}{2}\left(1-\frac{\delta}{2}\right)\right.$$

[illegible]

$K\left(\frac{de}{dt}\right)^2 = m v^2$ $\qquad\qquad n = \frac{v}{2\pi r} = \frac{5.10^4}{6.10^{-8}} = 10^{12}$

$m r^2$

$\cancel{\ast}\left(\frac{2\pi}{\tau}\right)^2 = \left(\frac{v}{r}\right)^2$

$\frac{2\pi}{\tau} = \frac{v}{r}$

$$\Sigma\, r^2 = 12 + 4\delta + \frac{1+\frac{\delta}{2}}{\sqrt{2}}[X] + \frac{1}{\sqrt{2}}[Y] - \frac{\delta}{2\sqrt{2}}\begin{pmatrix} y_2 + y_9 - y_1 - y_{10} \\ + z_4 + z_{12} - z_3 - z_{11} \end{pmatrix} + \frac{4}{\sqrt{2}}[Z]$$

$$+\, 6(x_0^2 + y_0^2 + z_0^2) - (x_0\, \Sigma_x + y_0\, \Sigma_y + z_0\, \Sigma_z) + \frac{\Sigma x^2 + \Sigma y^2 + \Sigma z^2}{2} \ast -$$

$$-\left(1 + \frac{3\delta}{2}\right)\left[2 x_0^2 - \frac{1}{2}\, x_0 \,\Sigma x + \frac{\Sigma x^2}{4}\right] -$$

$$-\left(1 - \frac{\delta}{2}\right)\left[y_0^2 - \frac{1}{2}\, y_0 (y_1 + y_2 + \right.$$

$$X = \frac{4\varphi}{\sqrt{2}}$$

$$U = A + B(x_0, y_0, z_0) + C(x_0^2, y_0^2, z_0^2)$$

$$\int (A + Bx + Cx^2)\, e^{-h(A+Bx+Cx^2)}\, dx = e^{-hA}\, e^{\frac{h^2B^2}{4C}} \sqrt{\frac{\pi}{C}} \left[A + \frac{B^2 h}{2C} + C\left(\frac{1}{2C} + \frac{h^2B^2}{4C^2}\right)\right.$$

$$= e^{-hA}\, e^{\frac{h^2B^2}{4hC}} \sqrt{\frac{\pi}{hC}} \left[A - \frac{B^2 h}{2hC} + C\left(\frac{1}{2hC} + \frac{h^2B^2}{4h^2C^2}\right)\right]$$

$$= e^{-hA + \frac{B^2 h}{4C}} \sqrt{\frac{\pi}{hC}} \left[A - \frac{B^2}{2C} + \frac{1}{2h} + \frac{B^2}{4C}\right]$$

$$= \cancel{e^{-hA + \frac{B^2h}{4C}}}$$

$$= e^{-h\left(A - \frac{B^2}{4C}\right)} \sqrt{\frac{\pi}{hC}} \left[A - \frac{B^2}{4C} + \frac{1}{2h}\right]$$

$$\int e^{-h(A+Bx+Cx^2)}\, dx = e^{-h\left(A - \frac{B^2}{4C}\right)} \sqrt{\frac{\pi}{hC}}$$

$$U = A + B_1 x + B_2 y + B_3 z + C_1 x^2 + C_2 y^2 + C_3 z^2$$

$$\int U e^{-hU}\, dx = \sqrt{\frac{\pi}{hC_1}}\; e^{-h\left[A + B_2 y + B_3 z + C_2 y^2 + C_3 z^2 - \frac{B_1^2}{4C_1}\right]}$$

$$\left[\frac{1}{2h} + A + B_2 y + B_3 z + \qquad \right]$$

$$\int \cdots\, dy = \sqrt{\frac{\pi^2}{h^2 C_1 C_2}}\; e^{-h\left[A + B_3 z + C_3 z^2 - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2}\right]}$$

$$\left\{ \frac{2}{2h} + A + B_3 z + C_3 z^2 - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2} \right\}$$

$$\int \cdots\, dz = \sqrt{\frac{\pi^3}{h^3 C_1 C_2 C_3}}\; e^{-h\left[A - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2} - \frac{B_3^2}{4C_3}\right]} \left[\frac{3}{2h} + A - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2} - \frac{B_3^2}{4C_3}\right]$$

$$\sum r^2 = \overset{12+8\delta}{\cancel{\quad}} + (1+\delta)\sqrt{2}\left[x_1+x_2-x_9-x_{10}+x_3+x_4-x_{11}-x_{12}\right] +$$

$$+\sqrt{2}\left[y_2+y_9+y_6+y_8-y_1-y_{10}-y_5-y_7\right] +$$

$$+\sqrt{2}\left[z_4+z_5+z_6+z_{12}-z_3-z_7-z_9-z_{11}\right] +$$

$$+ 12\,(x_0^2+y_0^2+z_0^2) - 2(x_0\sum x + y_0 \sum y + z_0 \sum z) + \sum x^2 + \sum y^2 + \sum z^2$$

$$r_1 = 1+\frac{\delta}{2} - \frac{x_0-x_1}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{y_0-y_1}{\sqrt{2}}\left(1-\frac{\delta}{2}\right) + \frac{(x_0-x_1)^2+\cdots}{2}\left(1-\frac{\delta}{2}\right) - \frac{(x_0-x_1)^2}{4}\left(1+\frac{\delta}{2}\right)$$

$$- \frac{(y_0-y_1)^2}{4}\left(1-\frac{3\delta}{2}\right) + \frac{(x_0-x_1)(y_0-y_1)}{2}\left(1-\frac{\delta}{2}\right)$$

$$\cancel{\sum r = 12 + 2\delta + \frac{1+\frac{\delta}{2}}{\sqrt{2}}[X] + \frac{1-\frac{\delta}{2}}{\sqrt{2}}[Y]}$$

$$\sum_{1\ldots10} r = 4\left(1+\frac{\delta}{2}\right) + \frac{1+\frac{\delta}{2}}{\sqrt{2}}\sum_{1\,2\,9\,10} X + \frac{1-\frac{\delta}{2}}{\sqrt{2}}\sum_{2\,9\,1\,10} Y + \frac{\sum(x_0-x_1)^2\cdots}{2}\left(1-\frac{\delta}{2}\right)$$

$$- \sum\frac{(x_0-x_1)^2}{4}\left(1+\frac{3\delta}{2}\right) - \frac{\sum(y_0-y_1)^2}{4}\left(1-\frac{\delta}{2}\right) + \frac{\left(1-\frac{\delta}{2}\right)}{2}\Big[(x_0-x_1)(y_0-y_1)-(x_0-x_2)(y_0-y_2)$$

$$- \sum(z_0-z_1)^2\cdot\frac{\delta}{4} \qquad + (x_0-x_9)(y_0-y_9)-(x_0-x_{10})(y_0-y_{10})\Big]$$

$$\sum_{3\ldots12} r = 4\left(1+\frac{\delta}{2}\right) + \frac{1+\frac{\delta}{2}}{\sqrt{2}}\sum_{3\ldots} X + \frac{\left(1-\frac{\delta}{2}\right)}{\sqrt{2}}\sum Z + \sum\frac{\cdots}{2}\left(1-\frac{\delta}{2}\right)$$

$$- \sum\frac{(x_0-x_3)^2}{4}\left(1+\frac{3\delta}{2}\right) - \frac{\sum(z_0-z_3)^2}{4}\left(1-\frac{\delta}{2}\right) + \frac{1-\frac{\delta}{2}}{2}\Big[(x_0-x_3)(z_0-z_3)-(x_0-x_4)(z_0-z_4)$$

$$- \sum(y_0-y_3)^2\frac{\delta}{4} \qquad + (x_0-x_{12})(z_0-z_{12})-(x_0-x_{11})(z_0-z_{11})\Big]$$

$$\sum_{5\ldots8} r = 4 + \cancel{\frac{1+\frac{\delta}{2}}{\sqrt{2}}\sum Z} \frac{1}{\sqrt{2}}\sum Z + \frac{1}{\sqrt{2}}\sum Y + \frac{\sum(x_0-x)^2\cdots}{2}$$

$$- \sum\frac{(z_0-z_5)^2}{4} - \frac{\sum(y_0-y_5)^2}{4} + \frac{1}{2}\Big[(y_0-y_5)(z_0-z_5) - (y_0-y_7)(z_0-z_7)$$

$$+ (y_0-y_8)(z_0-z_8) - (y_0-y_6)(z_0-z_6)\Big]$$

$$\cancel{r_{02}^2 = 1+\delta - (1+\delta)(x_0 - x_2)\sqrt{2} - (y_0 - y_2)\sqrt{2} + (x_0-x_2)^2 + (y_0-y_2)^2 + (z_0-z_2)^2}$$

$$r_{01}^2 = 1+\delta - (1+\delta)(x_0 - x_1)\sqrt{2} + (y_0 - y_1)\sqrt{2} + (x_0-x_1)^2 + (y_0-y_1)^2 + (z_0-z_1)^2$$

$$r_{02}^2 = 1+\delta - (1+\delta)(x_0 - x_2)\sqrt{2} - (y_0 - y_2)\sqrt{2} + (x_0-x_2)^2 + (y_0-y_2)^2 + (z_0-z_2)^2$$

$r_{0,9}$ $\quad + \quad -$

$r_{0,10}$ $\quad + \quad +$

$$r_{1,2,9,10}^2 = 4(1+\delta) + (1+\delta)(x_1 + x_2 - x_9 - x_{10})\sqrt{2} + (+y_2 + y_9 - y_1 - y_{10})\sqrt{2}$$
$$+4(x_0^2 + y_0^2 + z_0^2) - 2x_0 \Sigma x + \Sigma x^2$$
$$\cancel{+4z} \quad -2y_0 \Sigma y + \Sigma y^2$$
$$-2z_0 \Sigma z + \Sigma z^2$$

$$r_3^2 = 1+\delta - (1+\delta)(x_0 - x_3)\sqrt{2} + (z_0 - z_3)\sqrt{2} +$$

$$r_{3,4,11,12}^2 = 4(1+\delta) + (1+\delta)(x_3 + x_4 - x_{11} - x_{12})\sqrt{2} + (z_4 + z_{12} - z_3 - z_{11})\sqrt{2}$$
$$+4(x_0^2 + y_0^2 + z_0^2) - 2x_0 \Sigma x + \Sigma x^2$$
$$-2y_0 \Sigma y + \Sigma y^2$$
$$-2z_0 \Sigma z + \Sigma z^2$$

$$r_{5,7,6,8}^2 = 4 + (y_6 + y_8 - y_5 - y_7)\sqrt{2} + (z_5 + z_6 - z_7 - z_8)\sqrt{2}$$
$$+4(x_0^2 + y_0^2 + z_0^2) - 2x_0 \Sigma x$$
$$-2y_0 \ldots \quad -$$
$$- \quad -$$

$$\int_{-\infty}^{+\infty} y\, e^{-axy-by^2}\, dy = -\frac{ax}{2b^{3/2}}\, e^{\frac{a^2x^2}{4b}} \sqrt{\pi} \qquad a = -h(\varphi-\varphi'-\varphi'')\beta$$

$$b = 2h(2\varphi+\varphi')$$

$$\int_{-\infty}^{+\infty} e^{-axy-by^2}\, dy = \frac{e^{\frac{a^2x^2}{4b}}}{\sqrt{b}} \sqrt{\pi}$$

$$\frac{\partial}{\partial b}: \quad -\int_{-\infty}^{+\infty} y^2 e^{-axy-by^2}\, dy = e^{\frac{a^2x^2}{4b}} \sqrt{\frac{\pi}{b}} \left[-\frac{a^2x^2}{4b^2} - \frac{1}{2b}\right]$$

$$\overline{X}_y = (3\varphi+\varphi')\frac{\beta}{\sqrt{2}} - \cancel{\frac{\varphi-\varphi'-\varphi''}{\sqrt{2}} \frac{h(\varphi-\varphi'-\varphi'')\beta}{4h(2\varphi+\varphi')}} - \left[\frac{3}{4\sqrt{2}}(\varphi-\varphi'-2\varphi'')\beta + \right.$$

$$\left. \frac{5}{4\sqrt{2}}(\varphi-\varphi')\beta - \frac{1}{\sqrt{2}}(\varphi-\varphi'+\varphi'')\beta\right] \frac{4h(2\varphi+\varphi') \quad \frac{(\varphi-\varphi'-\varphi'')^2}{\sqrt{2}}h\beta}{h^2\left[16(2\varphi+\varphi')^2 - (\varphi-\varphi'-\varphi'')^2\beta^2\right]}$$

$$= \frac{\beta}{4\sqrt{2}} \frac{(12\varphi+4\varphi')(2\varphi+\varphi') - \cancel{(\varphi-\varphi'-\varphi'')^2}}{2\varphi+\varphi'} - \frac{\beta}{4\sqrt{2}}\left[3\varphi - 3\varphi' - 6\varphi'' + 5\varphi - 5\varphi'\right.$$

$$\left. -4\varphi + 4\varphi' - 4\varphi''\right] \frac{1}{4h(2\varphi+\varphi')} - \frac{(\varphi-\varphi'-\varphi'')^2\beta}{16h\sqrt{2}(2\varphi+\varphi')^2}$$

$$= \frac{\beta}{4\sqrt{2}}\left\{4(3\varphi+\varphi') - \cancel{\frac{(\varphi-\varphi'-\varphi'')^2}{2\varphi+\varphi'}} - \frac{(4\varphi-4\varphi'-10\varphi'')}{4h(2\varphi+\varphi')}\right\}$$

$$\int e^{-b(y^2+z^2)}\,dy\,dz \underbrace{\int x^2 e^{-axy-bx^2}\,dx}$$

$$= e^{+\frac{a^2y^2}{4b}} \int x^2 e^{-\left(x\sqrt{b}+\frac{ay}{2\sqrt{b}}\right)^2} dx$$

$$= \frac{e^{+\frac{a^2y^2}{4b}}}{b^{3/2}} \int \left[\left(x\sqrt{b}+\frac{ay}{2\sqrt{b}}\right)^2 - ayx - \frac{a^2y^2}{4b}\right] e^{-(x\sqrt{b}+\ldots)^2} d(x\sqrt{\ldots})$$

$$= \frac{e^{+\frac{a^2y^2}{4b}}}{b^{3/2}} \left[\int z^2 e^{-z^2} dz - \cancel{\frac{ay}{\sqrt{b}} \int x\, e^{-x^2b-ayx} dx} - \cancel{\frac{a^2y^2}{4b} \int e^{-x^2b-ayx} dx}\right.$$

$$\left. - \frac{ay}{\sqrt{b}} \left[\cancel{\int z\, e^{-z^2} dz} \underbrace{- \frac{ay}{2\sqrt{b}} \int e^{-z^2} dz + \frac{ay}{4\sqrt{b}} \int e^{-z^2} dz}_{-\frac{ay}{4\sqrt{b}}\sqrt{\pi}}\right]\right.$$

$$= \frac{e^{+\frac{a^2y^2}{4b}}}{b^{3/2}} \left[\frac{\sqrt{\pi}}{2} + \frac{a^2y^2}{4b}\sqrt{\pi}\right] \text{stimmt}$$

$$= \frac{\int e^{-bz^2} dz \cdot \frac{1}{2b}\sqrt{\frac{\pi}{b}} \int e^{-by^2} e^{\frac{a^2y^2}{4b}} \left(1 + \frac{a^2y^2}{2b}\right) dy}{\int e^{-bz^2} dz \; \sqrt{\frac{\pi}{b}} \int e^{-\left(b-\frac{a^2}{4b}\right)y^2} dy} =$$

$$= \frac{1}{2b}\left\{1 + \frac{a^2}{2b} \frac{\int y^2 e^{-\left(b-\frac{a^2}{4b}\right)y^2} dy}{\int e^{-\left(b-\frac{a^2}{4b}\right)y^2} dy}\right\} = \frac{1}{2b}\left\{1 + \frac{a^2}{2b} \frac{1}{2\left(b-\frac{a^2}{4b}\right)}\right\}$$

$$= \frac{1}{2b} \frac{4b^2}{4b^2-a^2} = \frac{2b}{4b^2-a^2}$$

$$X_y = 2\varphi\left\{\frac{3\beta}{2\sqrt2} + \frac{\beta y}{4} - \frac{3x}{2} - \frac{yx}{2\sqrt2} - \frac{3x^2\beta}{8\sqrt2} - \frac{5y^2\beta}{8\sqrt2} + \frac{z^2\beta}{2\sqrt2}\right\}$$

$$-2\varphi'\left\{-\frac{\beta}{2\sqrt2} + \frac{\beta y}{4} + \frac{x}{2} - \frac{xy}{2\sqrt2} - \frac{3x^2\beta}{8\sqrt2} - \frac{5y^2\beta}{8\sqrt2} + \frac{z^2\beta}{2\sqrt2}\right\}$$

$$-\frac{\varphi''}{2}\left\{\beta y - \frac{2xy}{\sqrt2} - \frac{3x^2\beta}{\sqrt2} - \frac{2z^2\beta}{\sqrt2}\right\}$$

$$U = \Phi(t) + \varphi\left[-xy\beta + 4(x^2y^2z^2)\right]$$

$$+\varphi'\left[xy\beta + 2(x^2y^2z^2)\right]$$

$$+\varphi''\, xy\beta$$

$$\int e^{-axy - b(x^2+y^2+z^2)}\,dx\,dy\,dz = \int e^{-b(y^2+z^2)}\,dy\,dz \int e^{-axy - bx^2}\,dx$$

$$= \int e^{-b(y^2+z^2)}\,dy\,dz \int e^{-\left(x\sqrt b + \frac{ay}{2\sqrt b}\right)^2 + \frac{a^2y^2}{4b}}\,dx$$

$$= \cancel{e^{\frac{a^2}{4b}}\int e^{-b(y^2+z^2)}\,dy\,dz\,\sqrt{\frac{\pi}{b}}} = \sqrt{\frac{\pi}{b}}\int e^{-bz^2}\,dz \int e^{-\left(b - \frac{a^2}{4b}\right)y^2}\,dy$$

$$\frac{\int xy\, e^{\cdots}}{\int e^{\cdots}} = -\frac{a\sqrt\pi}{2b^{3/2}}\cdot\sqrt{\frac{b}{\pi}}\cdot\frac{1}{2\left(b - \frac{a^2}{4b}\right)} = -\frac{a}{4b^2 - a^2}$$

$$\overline{X}_y = (3\varphi + \varphi')\frac{\beta}{\sqrt{2}}$$

$$+ \frac{\int y\, e^{-b[xy + 4(x^2 + \dots)]}\, dx\,dy\,dz}{\int \dots\dots} + \int yx\, \frac{e^{-xy + \dots}}{\dots}$$

$$\int e^{-b\,4(x^2+z^2)}\, dx\,dz \underbrace{\int_{-\infty}^{+\infty} y\, e^{-bxy - 4by^2}\, dy}$$

$$\frac{-2\beta}{-2\beta}\int_{-\infty}^{+\infty} y\, e^{-\alpha y - \beta y^2}\, dy = \int \frac{-\alpha - 2\beta y + \alpha}{-2\beta}\, e^{-\alpha y - \beta y^2}\, dy$$

$$= -\frac{1}{2\beta}\left\{ \cancel{e^{-\alpha y - \beta y^2}\Big|} + \alpha \int_{-\infty}^{+\infty} e^{-\alpha y - \beta y^2}\, dy \right\} = -\frac{\alpha}{2\beta}\int_{-\infty}^{+\infty} e^{-\left(y\sqrt{\beta} + \frac{\alpha}{2\sqrt{\beta}}\right)^2 + \frac{\alpha^2}{4\beta}}\, dy$$

$$= -\frac{\alpha}{2\beta^{3/2}}\, e^{\frac{\alpha^2}{4\beta}}\sqrt{\pi}$$

$$\int \frac{y\, e^{-axy - b(x^2+y^2+z^2)}\, dx\,dy\,dz}{\int \dots} = -\int e^{-b(x^2+z^2)}\, \frac{ax}{2\, b^{3/2}}\, e^{\frac{a^2x^2}{4b}}\sqrt{\pi}\, dx\,dz = ()$$

$$\int \frac{xy\, e^{-axy - b(x^2+y^2+z^2)}\, dx\,dy\,dz}{\int \dots} = -\frac{a\sqrt{\pi}}{2b^{3/2}} \int e^{-bz^2}\, dz \int e^{-\left(b - \frac{a^2}{4b}\right)x^2} \cdot x^2\, dx$$

$$r = r_0 - \frac{x x_k + y y_k + z z_k}{r_0} + \frac{x^2+y^2+z^2}{2 r_0} - \frac{x^2 x_k^2}{2 r_0^3} - \frac{y^2 y_k^2}{2 r_0^3} - \frac{z^2 z_k^2}{2 r_0^3} - xy\frac{x_k y_k}{r_0^3} - \dots$$

$$\left[1 \pm \frac{\beta}{2} \mp \frac{x}{\sqrt{2}}\left(1 \pm \frac{\beta}{2}\right) - \frac{y}{\sqrt{2}}\left(1 \mp \frac{\beta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1 \mp \frac{\beta}{2}\right) - \frac{x^2}{4}\left(1 \pm \frac{\beta}{2}\right) - \frac{y^2}{4}\left(1 \mp \frac{3\beta}{2}\right) \mp \frac{xy}{2}\left(1 \mp \frac{\beta}{2}\right)\right] x$$

$$-\left(\frac{1+\beta}{\sqrt{2}}\right)\left[1 + \frac{\beta}{2} - \frac{x}{\sqrt{2}}\left(1+\frac{\beta}{2}\right) - \frac{y}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\beta}{2}\right) - \frac{xy}{2}\right.$$

$$+ \frac{1-\beta}{\sqrt{2}}\left[1 - \frac{\beta}{2} + \frac{x}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) - \frac{y}{\sqrt{2}}\left(1+\frac{\beta}{2}\right) + \dots \left(1+\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1-\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1+\frac{3\beta}{2}\right) + \frac{xy}{2}\right.$$

$$2x\left[\cancel{\frac{\beta}{2} - \frac{x}{\sqrt{2}} + \frac{y\beta}{2\sqrt{2}} + \frac{x^2+y^2+z^2}{2}} - \frac{x^2}{4} - \frac{y^2}{4}\right.$$

$$\cancel{2x\left[\frac{\beta}{2} - \frac{x}{\sqrt{2}} + \frac{y\beta}{2\sqrt{2}}\right] - \frac{x^2+y^2+z^2}{2}\frac{\beta}{4} - \frac{x^2\beta}{8} + y^2\frac{3\beta}{8} - \frac{xy}{2}}$$

$$\left\{ - \frac{3\beta}{\sqrt{2}} + \frac{x}{\sqrt{2}} + \frac{y\beta}{2} - \frac{x^2+y^2+z^2}{2}\frac{\beta}{\sqrt{2}} + \frac{x^2}{4}\frac{3\beta}{\sqrt{2}} - \frac{y^2}{4}\frac{\beta}{\sqrt{2}} + \frac{xy}{\sqrt{2}}\right.$$

$$+2x\left[1 - \frac{x\beta}{2\sqrt{2}} - \frac{y}{\sqrt{2}}\right]$$

$\frac{3}{4} - 1$

$$= \left\{-\frac{3\beta}{\sqrt{2}} + 3x + \frac{y\beta}{2} - \frac{x^2+y^2+z^2}{2}\frac{\beta}{\sqrt{2}} + \frac{x^2}{4}\frac{\beta}{\sqrt{2}} - \frac{y^2\beta}{4\sqrt{2}} - \frac{xy}{\sqrt{2}}\right.$$

$$\left[1 - \frac{\beta x}{\sqrt{2}} - \frac{y \mp z}{\sqrt{2}} + \frac{x^2+y^2+z^2}{2} - \frac{(y\mp z)^2}{4} - \frac{x(y\mp z)\beta}{2}\right]\left(x - \frac{\beta}{\sqrt{2}}\right)$$

$$+2\left\{x - \frac{\beta x^2}{\sqrt{2}} - \frac{yx}{\sqrt{2}} - \frac{\beta}{\sqrt{2}} + \frac{\beta y}{2} - \frac{x^2+y^2+z^2}{2}\frac{\beta}{\sqrt{2}} + \frac{(y^2+z^2)\beta}{4\sqrt{2}}\right\}$$

$$= \left\{-\frac{5\beta}{\sqrt{2}} + 5x - \frac{(x^2+y^2+z^2)3}{2}\frac{\beta}{\sqrt{2}} - \frac{x^2 9\beta}{4\sqrt{2}} - \frac{3yx}{\sqrt{2}} + \frac{z^2\beta}{2\sqrt{2}}\right\}$$

$$+\frac{3y\beta}{2} \qquad + \frac{y^2\beta}{4\sqrt{2}}$$

5

$$\sum r(x - x_k) = x \sum\left(r_0 - \frac{x x_k + y y_k + z z_k}{r_0}\right)$$

$$- \sum\Big[\underbrace{x_k r_0}_{\frac{5\rho}{\sqrt2}} - x\frac{x_k^2}{r_0} + \frac{x^2+y^2+z^2}{2}\frac{x_k}{r_0} - \frac{x^2}{2}\frac{x_k^3}{r_0^3} - \frac{y^2}{2}\frac{x_k y_k^2}{r_0^3} - \frac{z^2}{2}\frac{x_k z_k^2}{r_0^3}$$

$$- y\sum\frac{x_k y_k}{r_0} - xy\sum\frac{x_k^2 y_k}{r_0^3} - \cancel{xz\sum\frac{x_k^2 z_k}{r_0^3}} - \cancel{yz}\Big]$$

$$= -\sum x_k r_0 + x\underbrace{\sum\left(r_0 + \frac{x_k^2}{r_0}\right)}_{=5} - x^2\sum\left(\frac{x_k}{r_0} + \frac{1}{2}\frac{x_k^3}{r_0^3}\right) + \frac{y^2}{2}\underbrace{\sum\frac{x_k y_k^2}{r_0^3}}_{\frac{\rho}{2\sqrt2}} + \frac{z^2}{2}\underbrace{\cancel{\sum\frac{x_k z_k^2}{r_0^3}}}_{=\frac{\rho}{\sqrt2}}$$

$$- \frac{x^2+y^2+z^2}{2}\sum\frac{x_k}{r_0} - xy\left(\sum\frac{y_k}{r_0} - \frac{x_k^2 y_k}{r_0^3}\right) - \cancel{xz\sum\frac{z_k}{r_0}} + y\left[\rho + \frac{1+\rho}{2}(1-\rho_2) - \frac{1-\rho}{2}(1+\rho_2)\right]$$

$$= -\frac{5\rho}{\sqrt2} + 5x - x^2\frac{9\rho}{4\sqrt2} - \frac{x^2+y^2+z^2}{2}\frac{3\rho}{\sqrt2} - xy\frac{4}{\sqrt2} + y\cdot\frac{3\rho}{2}$$

$$+ y^2\frac{\rho}{4\sqrt2} + \frac{xy}{\sqrt2}\underbrace{\left(\frac{1+2\rho}{2}\left(1-\frac{3\rho}{2}\right) + \frac{1-2\rho}{2}\left(1+\frac{3\rho}{2}\right)\right)}_{=\frac{1}{\sqrt2}}$$

$$\frac{3\rho}{\sqrt2}\cdot\frac{3\rho}{4\sqrt2}$$

$$= -\frac{5\rho}{\sqrt2} + 5x - x^2\cdot\frac{15\rho}{4\sqrt2} + \frac{3y\rho}{2} - y^2\frac{5\rho}{4\sqrt2} - z^2\frac{\rho}{\sqrt2} - \cancel{\frac{\rho}{\sqrt2}} - xy\frac{3}{\sqrt2}$$

stimmt

$$+\left\{-\frac{3\rho}{\sqrt2} + 3x - \frac{y\rho}{2} + \frac{3x^2\rho}{4\sqrt2} + \frac{5y^2\rho}{4\sqrt2} - \frac{z^2\rho}{\sqrt2} + \frac{xy}{\sqrt2}\right\}$$

$$-4\left\{-\rho\sqrt2 + 2x\right\}$$

$$= y\rho - \frac{3x^2\rho}{\sqrt2} - \frac{2z^2\rho}{\sqrt2} - xy\frac{2}{\sqrt2}$$

$$\frac{\partial}{\partial x}\sum (r-1)^3 = 3\left\{\rho\left[y - \frac{2z^2}{\sqrt2} - \frac{3x^2}{4\sqrt2}\right] - 2\frac{xy}{\sqrt2}\right\}$$

$$= -\frac{5\beta}{\sqrt2} + 3x - \frac{3y\beta}{2} + \frac{3x^2\beta}{4\sqrt2} + \frac{xy}{\sqrt2} + \frac{x^2+y^2+z^2}{2}\frac{3\beta}{\sqrt2} - \frac{3y^2\beta}{4\sqrt2}$$

$$= -\frac{5\beta}{\sqrt2} + 3x - \frac{3y\beta}{2} + \frac{9\beta}{4\sqrt2}x^2 + y^2\frac{3\beta}{4\sqrt2} + \frac{z^2\,3\beta}{2\sqrt2} + \frac{xy}{\sqrt2}$$

$$\sum_{9}(r_2-1)^3 = 2\left\{-\frac{y^3}{2\sqrt2} + \frac{3\beta}{2}\left[xy + \frac{xy^2}{2\sqrt2} - \frac{xz^2}{\sqrt2} - \frac{x^3}{2\sqrt2}\right]\right\}$$

$$-\frac{3x^2y}{2\sqrt2}$$

$$\cancel{\beta=0 \quad x \ldots z \quad y \ldots\ldots (x+y)}$$

$\beta = 0$

$x \ldots \beta x$

$y \ldots y+z$

$$\sum_{8}(r_6-1)^3 = 2\left\{-\frac{(y\pm z)^3}{2\sqrt2} \ldots + \frac{x(y\pm z)^2}{4\sqrt2}\right\}$$

$$-\frac{3}{2\sqrt2}\beta x(y\pm z)^2$$

$$= 2\left\{-\frac{y^3+3yz^2}{2\sqrt2} + \frac{3\beta}{2}\left[\frac{y^2+z^2}{2} - \frac{y(x^2y^2z^2)}{2\sqrt2} + \frac{3(y^3+3yz^2)}{4\sqrt2}\right]\right\}$$

$$-\frac{3x^2y}{2\sqrt2}$$

$$\cancel{\frac{y^3}{\sqrt2}} + 4yz^2 - 2yx^2$$

$$\sum_{4}(r-1)^3 = 2\left\{ - \frac{y^3}{\sqrt2} - \frac{3yz^2}{2\sqrt2} + \frac{3\beta}{2}\left[xy + \frac{xy^2}{\sqrt2} - \frac{xz^2}{\sqrt2} - \frac{x^3}{2\sqrt2}\right]\right\}$$

$$\frac{\partial}{\partial x} = 3\beta\left[y - \frac{z^2}{4\sqrt2} - \frac{3x^2}{8\sqrt2}\right] - \frac{3xy}{\sqrt2} - \frac{xy^2+z^2}{\sqrt2}$$

$$\frac{\partial}{\partial x}\sum(r-1)^3 = 3\sum r^2\frac{\partial r}{\partial x} - 3\frac{\partial}{\partial x}\sum r^2 + 3\frac{\partial}{\partial x}\sum r$$

$$4\sqrt2 - \frac{5}{\sqrt2} - \frac{3}{\sqrt2}$$

$$\cancel{\sum(r^2-2r+1) = \sum(r-1)^2 + 2\sum(r-1)}$$

$$= 3\sum r^2\frac{\partial r}{\partial x} - 3\frac{\partial}{\partial x}\sum(r-1)^2 - 3\frac{\partial}{\partial x}\sum r$$

$$-\frac{5\beta}{\sqrt2} + 3x - \frac{3y\beta}{2} + \frac{9\beta}{4\sqrt2}x^2 + \frac{3\beta}{4\sqrt2}y^2 + \frac{3\beta}{2\sqrt2}z^2 + \frac{xy}{\sqrt2} - 4\left\{-\beta\sqrt2 + 2x\right\}$$

$$+2\left\{-\frac{3\beta}{2\sqrt2} + \frac{3x}{2} - \frac{y\beta}{4} + \frac{3x^2\beta}{8\sqrt2} + \frac{5y^2\beta}{8\sqrt2} - \frac{z^2\beta}{2\sqrt2} + \frac{xy}{2\sqrt2}\right\}$$

$$\frac{\partial}{\partial x}\sum(r-1)^2 = \frac{\partial}{\partial x}\sum r^2 - 2\,\frac{\partial \sum r}{\partial x}$$

$$\sum r^2 = 4\left[1-\beta x\sqrt{2} - y\sqrt{2} + x^2+y^2+z^2\right]$$

$$+\cancel{\left[1-\beta x\sqrt{2} - y\sqrt{2} + z^2\right]}$$

$$\frac{\partial}{\partial x}\sum(r-1)^2 = 4\left\{-\beta\sqrt{2}+2x\right\} - 4\left\{-\frac{3\beta}{2\sqrt{2}} - \frac{\beta y}{4} + \frac{3x}{2} + \frac{xy}{2\sqrt{2}} + \frac{3x^2\beta}{8\sqrt{2}} + \frac{5y^2\beta}{8\sqrt{2}} - \frac{z^2\beta}{2\sqrt{2}}\right\}$$

$$= 4\left\{\frac{-\beta}{2\sqrt{2}} + \frac{x}{2} + \frac{\beta y}{4} - \frac{xy}{2\sqrt{2}} - \frac{3x^2\beta}{8\sqrt{2}} - \frac{5y^2\beta}{8\sqrt{2}} + \frac{z^2\beta}{2\sqrt{2}}\right\}$$ Stimmt

$$\sum(x-x_k)r_k = -\sum x_k r_0 + x\sum\left(r_0 - \frac{x_k^2}{r_0}\right) - y\sum\frac{x_k y_k}{r_0} + x^2\sum\left(\frac{x_k}{r_0} - \frac{3}{2}\frac{x_k^3}{r_0^3}\right) +$$

$$\cancel{y^2\sum\left(\frac{x_k}{r_0} - \frac{3}{2}\frac{y_k^2}{r_0^3}\right)} - \;+ xy\sum\left(\frac{y_k}{r_0} - 3\frac{x_k^2 y_k}{r_0^3}\right) + \frac{x^2+y^2+z^2}{2}\sum\frac{x_k}{r_0} -$$

$$-\frac{3}{2}y^2\sum\frac{x_k y_k^2}{r_0^3} - \cancel{\frac{3}{2}z^2\sum\frac{x_k z_k^2}{r_0^3}}$$

$$= -\left[\frac{2\beta}{\sqrt{2}} + \frac{1+\beta}{\sqrt{2}}\left(1+\frac{\beta}{2}\right) - \frac{1-\beta}{\sqrt{2}}\left(1-\frac{\beta}{2}\right)\right] + x\left[4 - \frac{1+2\beta}{2}\left(1-\frac{\beta}{2}\right) + \frac{1-2\beta}{2}\left(1+\frac{\beta}{2}\right)\right]$$

$$-y\left[\beta + \frac{1}{\sqrt{2}}\left(\frac{1+\beta}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) - \frac{1-\beta}{\sqrt{2}}\left(1+\frac{\beta}{2}\right)\right)\right] + x^2\left[\frac{2\beta}{\sqrt{2}} + \frac{1+\beta}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) - \frac{1-\beta}{\sqrt{2}}\left(1+\frac{\beta}{2}\right)\right] -$$

$$-\frac{3}{2}\left(\frac{1+3\beta}{2\sqrt{2}}\left(1-\frac{3\beta}{2}\right) - \frac{1-3\beta}{2\sqrt{2}}\left(1+\frac{3\beta}{2}\right)\right) + xy\left[\frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}}\left(\frac{1+2\beta}{2}\left(1-\frac{3\beta}{2}\right) + \frac{1-2\beta}{2}\left(1+\frac{3\beta}{2}\right)\right)\right]$$

$$+\frac{x^2+y^2+z^2}{2}\left[\frac{2\beta}{\sqrt{2}} + \frac{1+\beta}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) - \frac{1-\beta}{\sqrt{2}}\left(1+\frac{\beta}{2}\right)\right] - \frac{3y^2}{2}\,\frac{1}{2}\left[\frac{2\beta}{\sqrt{2}} + \frac{1+\beta}{\sqrt{2}}\left(1-\frac{3\beta}{2}\right) - \frac{1-\beta}{\sqrt{2}}\left(1+\frac{3\beta}{2}\right)\right]$$

$$= -\frac{5\beta}{\sqrt{2}} + 3x - \frac{3y\beta}{2} + \left[\frac{x^2\,3\beta}{\sqrt{2}} - \frac{9\beta}{4}\right] + \frac{xy}{\sqrt{2}} +$$

$$+\frac{x^2+y^2+z^2}{2}\,\frac{3\beta}{\sqrt{2}} - \frac{3y^2}{4}\,\frac{\beta}{\sqrt{2}}$$

$$\sum \frac{x-x_k}{r} = \frac{\ }{r_0} - x = \sum (x-x_k)\left[\frac{1}{r_0} + \frac{x x_k + y y_k + \cancel{z z_k}}{r_0^2} - \frac{x^2 y^2 z^2}{2 r_0^3} + \frac{3}{2}\frac{x^2 x_k^2 + y^2 y_k^2 + z^2 z_k^2}{r_0^5}\right.$$

$$\left. + \frac{3(x y\, x_k y_k + x z\, x_k z_k + y z\, y_k z_k)}{r_0^5}\right]$$

$$= x \sum \frac{1}{r_0} + x^2 \sum \frac{x_k}{r_0^3} + xy \sum \frac{y_k}{r_0^3} - \cancel{\frac{x(x^2 y^2 z^2)}{2} \sum \frac{1}{r_0^3}} + \frac{3}{2}\left[\cancel{x^3 \sum \frac{x_k^2}{r_0^5} + x y^2 \sum \frac{y_k^2}{r_0^5} + x z^2 \sum \frac{z_k^2}{r_0^5}}\right.$$

$$\cancel{+ 3\left(x^2 y \sum x_k y_k + x^2 z \sum x_k z_k + x y z \sum y_k z_k\right)} -$$

$$- \left\{ \sum \frac{x_k}{r_0} + x \sum \frac{x_k^2}{r_0^3} + y \sum \frac{x_k y_k}{r_0^3} + z \sum \cancel{\frac{x_k z_k}{r_0^2}} - \frac{x^2 y^2 z^2}{2} \sum \frac{x_k}{r_0^3} + \frac{3}{2} x^2 \sum \frac{x_k^3}{r_0^5}\right.$$

$$\left. + \frac{3}{2} y^2 \sum \frac{x_k y_k^2}{r_0^5} + \frac{3}{2} z^2 \sum \frac{x_k z_k^2}{r_0^5} + 3xy \sum \frac{x_k^2 y_k}{r_0^5} + 3xz \sum \cancel{\frac{x_k^2 z_k}{r_0^5}} + 3yz \sum \cancel{\frac{x_k y_k z_k}{r_0^5}}\right\}$$

$$= \cdot 4x + x^2\left[\frac{2\beta}{\sqrt2} + \cancel{\frac{1+\beta}{\sqrt2}\left(1-\frac{3\beta}{2}\right) - \frac{1-\beta}{\sqrt2}\left(1+\frac{3\beta}{2}\right)}\right] + xy \frac{4}{\sqrt2} - \frac{x(x^2 y^2 z^2)}{2}\cdot 4 +$$

$$+ \frac{3x^3}{2}\left[\underbrace{\cancel{\frac{1+2\beta}{2}\left(1-\frac{5\beta}{2}\right) + \frac{1-2\beta}{2}\left(1+\frac{5\beta}{2}\right)}}_{=1}\right] + \cancel{\frac{3xy^2}{2}} + \frac{3}{2} x z^2 + 3x^2 y\left[\beta+\beta\right]$$

$$- \left[\frac{3\beta\sqrt2}{2} + \cancel{\frac{1+\beta}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{1-\beta}{\sqrt2}\left(1+\frac{\beta}{2}\right)}\right] - x\left[\overset{=1}{\frac{1+2\beta}{2}\left(1-\frac{3\beta}{2}\right) + \frac{1-2\beta}{2}\left(1+\frac{3\beta}{2}\right)}\right]$$

$$-y\left[\beta + \frac{1+\beta}{2}\left(1-\frac{3\beta}{2}\right) - \frac{1-\beta}{2}\left(1+\frac{3\beta}{2}\right)\right] + \frac{x^2 y^2 z^2}{2}\left[\frac{\beta\sqrt2}{4} + \cancel{\frac{1+\beta}{\sqrt2}\left(1-\frac{3\beta}{2}\right) - \frac{1-\beta}{\sqrt2}\left(1+\frac{3\beta}{2}\right)}\right]$$

$$- \frac{3x^2}{2}\left[\overset{\frac{\beta}{2\sqrt2}}{\cancel{\frac{1+3\beta}{2\sqrt2}\left(1-\frac{5\beta}{2}\right) - \frac{1-3\beta}{2\sqrt2}\left(1+\frac{5\beta}{2}\right)}}\right] - \frac{3}{2}y^2\left[\frac{\beta}{\sqrt2} + \underbrace{\cancel{\frac{1+\beta}{2\sqrt2}(1-5\beta/2) - \frac{1-\beta}{2\sqrt2}(1+5\beta/2)}}_{\frac{\beta}{\sqrt2}}\right] = \frac{\beta}{2\sqrt2}$$

$$1 - \frac{3\beta}{2} - \left(1+\frac{3\beta}{2}\right) = -\frac{3\beta}{2\sqrt2}$$

$$- \frac{3z^2}{2}\left[\frac{2\beta}{2\sqrt2}\right] + 3xy\left[\cancel{\frac{1+2\beta}{2\sqrt2}\left(1-\frac{5\beta}{2}\right) - \frac{1-2\beta}{2\sqrt2}\left(1+\frac{5\beta}{2}\right)}\right]$$

$$= -\frac{3\beta}{\sqrt2} + 3x - y\frac{\beta}{\sqrt2} + x^2\left[\frac{\beta}{\sqrt2} - \frac{3\beta}{4\sqrt2}\right]^{=\frac{1}{\sqrt2}} + \frac{3y^2\beta}{4\sqrt2} + xy\frac{1}{\sqrt2} + \frac{x^2 y^2 z^2 \beta}{2\sqrt2} - \frac{3z^2\beta}{2\sqrt2}$$

$$\cancel{2x(x^2 y^2 z^2) + \frac{3x^3}{2} + 1xy^2 + 6x^2 y\beta + \frac{3}{2}xz^2}$$

$$\frac{4-3+2}{4\sqrt2} = \frac{3}{4\sqrt2} \qquad 3+$$

$$(r_6-1)^3 = \cancel{} - \frac{(y\pm z)^3}{2\sqrt2} = -\frac{y^3+3z^2y}{2\sqrt2}$$

8

$$\begin{matrix}6\\7\end{matrix} = \qquad +$$

$$\Sigma = 0$$

$$\Sigma_{1\cdots12} = 6\rho.xy$$

$$\Sigma(r-1)^3 = 12 + 24\,(x^2y^2+z^2) + 9.xy.\rho - 36\,(1+x^2y^2z^2) + 36 - 3xy\rho$$
$$+12\,(x^2y^2z^2) - 12$$
$$= 6xy\rho$$

stimmt

$$\Sigma\frac{x-x_k}{r} = -\Sigma\frac{x_k}{r_0} + x\,\Sigma\left(\frac{1}{r_0}-\frac{x_k^2}{r_0^3}\right) - y\,\Sigma\frac{x_ky_k}{r_0^3} + x^2\underbrace{\Sigma\left(\frac{x_k}{r_0^3}-\frac{3}{2}\frac{x_k^3}{r_0^5}\right)}_{\rho/4\sqrt2} + xy\underbrace{\Sigma\left(\frac{y_k}{r_0^3}-3\frac{x_k^2y_k}{r_0^5}\right)}_{\frac{1}{\sqrt2}}$$

$$+\frac{x^2y^2+z^2}{2}\underbrace{\Sigma\frac{x_k}{r_0^3}}_{\rho/\sqrt2} - \frac{3}{2}y^2\,\Sigma\frac{x_ky_k^2}{r_0^5} - \frac{3}{2}z^2\,\Sigma\frac{x_kz_k^2}{r_0^5} = \rho/\sqrt2$$

$$\sum (r-1)^2 = \sum r^2 - 2\sum r + 12 =$$

$$= 12(1+x^2+y^2+z^2) - 2\left[12 - xy\beta + 4(x^2+y^2+z^2)\right] + 12 =$$

$$= \cancel{\;\;} + 4(x^2+y^2+z^2) + 2xy\beta$$ stimmt

$$4\left\{\frac{x^2y^2}{2} + \frac{y^2z^2}{2} + \frac{x^2z^2}{2} + xy\frac{\beta}{2}\right\} = \uparrow$$

$$(r_2-1)^3 = -\frac{x^3}{2\sqrt2}\left(1+\frac{3\beta}{2}\right) - \frac{y^3}{2\sqrt2}\left(1-\frac{3\beta}{2}\right) + \frac{3\beta}{2}\left[\frac{x^2}{2} + \frac{y^2}{2} + xy\frac{\beta}{2} - \frac{x(x^2+z^2)}{4\sqrt2} + \frac{x^3(\ldots)}{2\sqrt2}\right.$$

$$\cancel{-\frac{x(x^2+y^2+z^2)}{2\sqrt2} - \frac{y(x^2+y^2+z^2-\beta)}{2\sqrt2} + \frac{x^3(1+\beta)}{}} + \frac{xy^2}{2\sqrt2}(\ldots) + \frac{x^2y}{4\sqrt2}$$

$$-\frac{x^3}{2\sqrt2}\cdot\frac{3\beta}{2} \quad \left. -\frac{y(x^2+z^2)}{4\sqrt2} + \frac{yx^2}{2\sqrt2} + \frac{y^3}{2\sqrt2} + \frac{xy^2}{4\sqrt2}\right] - \frac{3x^2y}{2\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{3xy^2}{2\sqrt2}\left(1-\frac{\beta}{2}\right)$$

$$(r_9-1)^3 = \pm\frac{3\beta}{2}\left[\frac{x^2+y^2}{2} \pm xy \mp \frac{x^3}{4\sqrt2} + \frac{yx^2}{4\sqrt2} \mp \frac{xy^2}{2\sqrt2} + \frac{y^3}{4\sqrt2} \mp \frac{xz^2}{2\sqrt2} - \frac{yz^2}{2\sqrt2}\right]$$

$$\mp\frac{x^3}{2\sqrt2} - \frac{y^3}{2\sqrt2} \quad - - ——$$

$$= 2\left\{-\frac{y^3}{2\sqrt2} + \frac{3\beta}{2}\left[xy + \frac{xy^2}{2\sqrt2} - \frac{xz^2}{4\sqrt2} - \frac{x^3}{2\sqrt2}\right] - \frac{3x^2y}{2\sqrt2} + \frac{3xy^2}{4\sqrt2}\beta\right\}$$

$$(r_{10}-1)^3 = \mp\frac{x^3}{2\sqrt2} + \frac{y^3}{2\sqrt2} \mp \frac{3\beta}{2}\left[\frac{x^2+y^2}{2} \mp xy \mp \frac{3x^3}{4\sqrt2} - \frac{yx^2}{4\sqrt2} \pm \frac{xy^2}{4\sqrt2} - \frac{y^3}{4\sqrt2} \mp \frac{xz^2}{4\sqrt2} + \frac{yz^2}{2\sqrt2}\right]$$

$$= 2\left\{\frac{y^3}{2\sqrt2} + \frac{3\beta}{2}\left(xy + \frac{3x^3}{4\sqrt2} - \frac{xy^2}{4\sqrt2} + \frac{xz^2}{4\sqrt2}\right) + \frac{3x^2y}{2\sqrt2} - \frac{3xy^2\beta}{4\sqrt2}\right\}$$

$$\sum_{1,2,9,10} = 4\left\{\frac{3\beta\, xy}{2}\right\} = 6\beta\cdot xy$$

$$\sum(r_{\frac{2}{9}}-1)^2 = 2\Big\{-\frac{\beta x}{\sqrt2} + \frac{x^2+y^2}{2} - xy\frac{\beta}{2} + \frac{3x^3\beta}{4\sqrt2} - \frac{y^3}{2\sqrt2} + \frac{x^2y}{2\sqrt2} - \frac{xy^2\, 7\beta}{4\sqrt2} + \frac{xz^2\beta}{2\sqrt2} + \frac{yz^2}{\sqrt2}$$

$$(r_1-1)^2 = 1 + \frac{\beta x}{\sqrt2} \mp \frac{\beta y}{\sqrt2} + \frac{x^2}{2}\left(1 \mp \frac{5\beta}{4}\right) + \frac{y^2}{2}\left(1\pm\frac{3\beta}{4}\right) \mp xy\left(1\pm\frac{\beta}{2}\right) \mp z^2\frac{\beta}{2} \mp \frac{x^3}{2\sqrt2}\left(1\pm\frac{3\beta}{2}\right)$$

$$+\frac{y^3}{2\sqrt2}\left(1\pm\frac{\beta}{2}\right) - \frac{x^2y}{2\sqrt2}\left(1\mp\frac{3\beta}{2}\right) \pm \frac{xy^2}{2\sqrt2}\left(1\pm\frac{7\beta}{2}\right) \mp \frac{xz^2}{\sqrt2}\left(1\pm\frac{\beta}{2}\right) + \frac{yz^2}{\sqrt2}\left(1\pm\frac{3\beta}{2}\right)$$

$$\sum_{10} = 2\Big\{\frac{\beta x}{\sqrt2} + \frac{x^2+y^2}{2} - yx\frac{\beta}{2} - \frac{3x^3\beta}{4\sqrt2} + \frac{y^3}{2\sqrt2} - \frac{x^2y}{2\sqrt2} + \frac{7xy^2\beta}{4\sqrt2} - \frac{xz^2\beta}{4\sqrt2} + \frac{yz^2}{\sqrt2}$$

$$\sum_{\substack{1\\2\\9\\10}} = 4\left\{\frac{x^2+y^2}{2} - xy\frac{\beta}{2}\right\}$$

$$(r_{\frac{6}{8}}-1)^2 = \frac{(y\pm z)^2}{2}\left(\cancel{1+\beta x\sqrt2}\right) + \beta x(y\pm z)\left(\cancel{\quad}\right) - \frac{\beta x}{\sqrt2}(x^2+y^2+z^2) + \frac{\beta x}{\sqrt2}\frac{(y\pm z)^2}{2}$$

$$-\frac{(y\pm z)}{\sqrt2}\left(\cancel{1+\frac{2\beta x}{\sqrt2}}\right)(x^2+y^2+z^2) + \frac{(y\pm z)^3}{2\sqrt2}\left(\cancel{1+2\beta x\sqrt2}\right)$$

$$=2\Big\{\frac{y^2+z^2}{2}(1+\beta x\sqrt2) + \beta xy\left(\cancel{\quad}\right) - \frac{\beta x^3}{\sqrt2} - \frac{\beta x(y^2+z^2)}{2\sqrt2}$$

$$-y\frac{(x^2+y^2+z^2)}{\sqrt2} + \frac{y^3+3z^2y}{2\sqrt2}\Big\}$$

$$=2\Big\{\frac{y^2+z^2}{2} + xy\beta - \frac{\beta x^3}{\sqrt2} + \frac{\beta x(y^2+z^2)}{2\sqrt2} - y\frac{(x^2+y^2+z^2)}{\sqrt2} + \frac{y^3+3z^2y}{2\sqrt2}\Big\}$$

$$\tfrac{5}{7} \quad =2\Big\{\frac{y^2+z^2}{2} + xy\beta + \frac{\beta x^3}{\sqrt2} - \frac{\beta x(y^2+z^2)}{2\sqrt2} + y(\ldots) - \ldots\Big\}$$

$$=4\left\{\frac{y^2+z^2}{2}+xy\beta\right\}$$

$$\sum_{34\,11\,12} = 4\left\{\frac{x^2+z^2}{2}\right\}$$

$$r_2 = 1+\frac{\beta}{2} - \frac{x}{\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y}{\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{x^2+y^2-z^2}{2}\left(1-\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\beta}{2}\right) - \frac{xy}{2}\left(1-\frac{\beta}{2}\right)$$

$$+\frac{x(x^2+y^2-z^2)}{2\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y(x^2+y^2-z^2)}{2\sqrt2}\left(1-\frac{3\beta}{2}\right) - \frac{x^3}{4\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y^3}{4\sqrt2}\left(1-\frac{5\beta}{2}\right) - \frac{3x^2y}{4\sqrt2}\left(1-\frac{\beta}{2}\right)$$

$$-\frac{3xy^2}{4\sqrt2}\left(1-\frac{3\beta}{2}\right)$$

$$(r_2-1)^2 = \frac{x^2}{2}(1+\beta) + \frac{y^2}{2}(1-\beta) - \frac{\beta x}{\sqrt2} - \frac{\beta y}{\sqrt2} + \frac{x^2+y^2-z^2}{2}\beta - \frac{x^2\beta}{4} - \frac{y^2\beta}{4} - \frac{xy\beta}{2}$$

$$+\frac{x(x^2+y^2-z^2)\beta}{2\sqrt2} + \frac{y(x^2+y^2-z^2)\beta}{2\sqrt2} - \frac{x^3\beta}{4\sqrt2} - \frac{y^3\beta}{4\sqrt2} - \frac{3x^2y\beta}{4\sqrt2} - \frac{3xy^2\beta}{4\sqrt2}$$

$$+xy - \frac{x(x^2+y^2-z^2)}{\sqrt2} + \frac{x^3}{2\sqrt2}(1+\beta) + \frac{xy^2}{2\sqrt2}(1-\beta) + \frac{x^2y}{\sqrt2} \qquad 1-\frac{3}{4}+1+\frac{1}{2} \quad \frac{7}{4}$$

$$-\frac{y(x^2+y^2-z^2)}{\sqrt2}(1-\beta) + \frac{x^2y}{2\sqrt2} + \frac{y^3}{2\sqrt2}(1-\beta) + \frac{xy^2}{\sqrt2}(1-\beta)$$

$$= -\frac{\beta x}{\sqrt2} - \frac{\beta y}{\sqrt2} + \frac{x^2}{2}\Big(1+\underbrace{\beta+\frac{\beta}{2}-\frac{\beta}{4}}_{1+\frac{5\beta}{4}}\Big) + \frac{y^2}{2}\left(1-\beta+\frac{\beta}{2}-\frac{\beta}{4}\right) + xy\left(1-\frac{\beta}{2}\right) + \frac{z^2\beta}{2}$$

$$1-\frac{3\beta}{4}$$

$$-\frac{x(x^2+y^2-z^2)}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{y(x^2+y^2-z^2)}{\sqrt2}\left(1-\frac{3\beta}{2}\right) + \frac{x^3}{2\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{y^3}{2\sqrt2}\left(1-\frac{5\beta}{2}\right)$$

$$+\frac{x^2y}{\sqrt2}\left(\frac{3}{2} - \frac{3\beta}{4}\right) + \frac{xy^2}{\sqrt2}\left(\frac{3}{2} - \frac{3\beta}{2} - \frac{3\beta}{4}\right)$$

$$-\frac{1}{2}+\frac{3\beta}{4}$$

$$\frac{x^3}{\sqrt2}\left(-1+\frac{\beta}{2}+\frac{1}{2}+\frac{\beta}{4}\right) + \frac{y^3}{2\sqrt2}\left(1-\frac{5\beta}{2}-2+3\beta\right) + \frac{x^2y}{\sqrt2}\left(\frac{3}{2}-\frac{3\beta}{4}-1+\frac{3\beta}{2}\right)$$

$$\frac{1}{2}+\frac{3\beta}{4}$$

$$+\frac{xy^2}{\sqrt2}\left(\frac{3}{2} - \frac{9\beta}{4} - 1 + \frac{\beta}{2}\right) - \frac{xz^2}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{yz^2}{\sqrt2}\left(1-\frac{3\beta}{2}\right)$$

$$\underset{9}{r_2} = -\frac{\beta x}{\sqrt2} \mp \frac{\beta y}{\sqrt2} + \frac{x^2}{2}\left(1\pm\frac{5\beta}{4}\right) + \frac{y^2}{2}\left(1\mp\frac{3\beta}{4}\right) \pm xy\left(1\mp\frac{\beta}{2}\right) \pm \frac{z^2\beta}{2} \mp \frac{x^3}{2\sqrt2}\left(1\mp\frac{3\beta}{2}\right) - \frac{y^3}{2\sqrt2}\left(1\mp\frac{\beta}{2}\right)$$

$$+\frac{x^2y}{2\sqrt2}\left(1\pm\frac{3\beta}{2}\right) \pm \frac{xy^2}{2\sqrt2}\left(1\mp\frac{7\beta}{2}\right) \mp \frac{xz^2}{\sqrt2}\left(1\mp\frac{\beta}{2}\right) - \frac{yz^2}{\sqrt2}\left(1\mp\frac{3\beta}{2}\right)$$

$$\sum r^2 = 12\,(1 + x^2 + y^2 + z^2)$$

$$r^3 = r_0^3 + \frac{3(x^2+y^2+z^2)}{2} \sum r_0 + \frac{3x^2}{2} \sum \frac{x_k^2}{r_0} + \frac{3y^2}{2} \sum \frac{y_k^2}{r_0} + \frac{\sum z^2}{2} \sum \frac{z_k^2}{r_0}$$

$$+ 3xy \sum \frac{x_k y_k}{r_0} + \cdots$$

$$8\left[\frac{1-2\beta}{4}\left(1+\frac{\beta}{2}\right) + \frac{1+2\beta}{4}\left(1-\frac{\beta}{2}\right)\right] + \frac{4}{2} = \sum \frac{x_k^2}{r_0} = 4$$

$$1 - \cancel{\tfrac{3\beta}{2}} + 1 + \cancel{\tfrac{3\beta}{2}}$$

$$\frac{\sum y_k^2}{r_0} = 4$$

$$\sum \frac{z_k^2}{r_0} = 4$$

$$\sum \frac{x_k y_k}{r_0} = \frac{8}{4}\left\{-(1-\beta)\left(1+\frac{\beta}{2}\right) + (1+\beta)\left(1-\frac{\beta}{2}\right)\right\} + \frac{4}{2}\beta = 3\beta$$

$$-1 + \frac{\beta}{2} + 1 + \frac{\beta}{2}$$

$$\sum \frac{x_k z_k}{r_0} = 0$$

$$\sum y_k z_k = 0$$

$$\sum r^3 = 12\left[1 + \frac{3(x^2+y^2+z^2)}{2}\right] + 6(x^2+y^2+z^2) + 9xy\beta$$

$$= 12 + 24(x^2+y^2+z^2) + 9xy\beta$$

r_2
9

$$\frac{\partial \Sigma}{\partial x} = 2\left\{ -\frac{3\rho}{2\sqrt2} - \frac{\rho y}{4} + \cancel{\ldots} + \frac{3x}{2} + \frac{yx}{2\sqrt2} \cancel{\ldots} \right.$$

$$\left. + \frac{3xy\rho}{8\sqrt2} + \frac{5y\rho}{2\sqrt2} - \frac{z^2\rho}{2\sqrt2} \right)$$

$$\sum_{5678} r = 4\left\{ 1 - \rho\frac{xy}{2} + \frac{x^2+y^2+z^2}{2} - \frac{x^2+y^2+z^2}{4} \right\}$$

$$\sum_{34} r = 4\left\{ 1 - \cancel{\ldots} + \cancel{\ldots} + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{4} - \frac{z^2}{4} \right\} + \cancel{\ldots} + \frac{x(x^2+y^2+z^2)}{2\sqrt2} - \frac{2(x^2+y^2+z^2)}{2\sqrt2}$$

$$- \cancel{\frac{x^3}{4\sqrt2}} + \cancel{\frac{z^3}{4\sqrt2}} + \cancel{\frac{3x^2z}{4\sqrt2}} - \cancel{\frac{3xz^2}{4\sqrt2}}$$

$$\sum_{1-12} r = 12\left\{ 1 - \frac{xy\rho}{12} + \frac{x^2+y^2+z^2}{2} - \frac{x^2+y^2+z^2}{6} \right\}$$

$$= 12\left\{ 1 - \frac{xy\rho}{12} + \frac{x^2+y^2+z^2}{3} \right\} = 12 - xy\rho + (x^2+y^2+z^2)4$$

(strict)

$$\Sigma r = \Sigma\left\{ r_0 - \cancel{\frac{x x_k + y y_k + z z_k}{r_0}} + \frac{x^2+y^2+z^2}{2 r_0} - \frac{x^2 x_k^2}{2 r_0^3} - \frac{y^2 y_k^2}{2 r_0^3} - \frac{z^2 z_k^2}{2 r_0^3} \right.$$

$$\left. - \frac{xy\, x_k y_k}{r_0^3} - \cancel{\frac{xz\, x_k z_k}{r_0^3}} - \cancel{\frac{yz\, y_k z_k}{r_0^3}} \right\}$$

$$\Sigma r = 12 + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{2}\left[\frac{1-2\rho}{2}\left(1+\frac{3\rho}{2}\right) + \frac{1+2\rho}{2}\left(1-\frac{3\rho}{2}\right) + 2 \right] - \frac{y^2}{2}\,\frac{8}{2} - \frac{z^2}{2}\,\frac{8}{2} -$$

$$- \frac{xy}{2}\left[-(1-\rho)2\left(1+\frac{3\rho}{2}\right) + (1+\rho)2\left(1-\frac{3\rho}{2}\right) + \cancel{\ldots} \right]$$

$\rho - 1 \quad -\frac{3\rho}{2}$ $\qquad -(1+\frac{\rho}{2})$

$\rho + 1 \quad -\frac{3\rho}{2}$ $\qquad +(1-\frac{\rho}{2})$

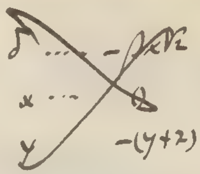
$$= 12\left(1 + \frac{x^2+y^2+z^2}{2}\right) - 2x^2 - 2y^2 - 2z^2 + \cancel{xy\rho}$$

$$= 12 + 6(x^2+y^2+z^2) - 2x^2 - 2y^2 - 2z^2 - xy\rho$$

$$= 12 + 4(x^2+y^2+z^2) - xy\rho$$

$$\Sigma \frac{x_k}{r_0} =$$

$$r_6 = \sqrt{1-\beta x\sqrt{2} - (y+2)\sqrt{2} + x^2 y^2 z^2}$$



$\delta > 0$

$x \to -\beta x$

$y \quad -(y+2)$

$$\frac{r}{8} = 1 - \frac{\beta x}{\sqrt{2}} - \frac{(y\pm 2)}{\sqrt{2}}\left([illegible] \right) + \frac{[illegible]}{\sqrt{2}}\left([illegible] \right) - \frac{(y\pm 2)^2}{4}\left(1 + \frac{3\beta x\sqrt{2}}{2}\right) \pm \frac{\beta x\sqrt{2}(y+2)}{2}$$

$$+ \frac{(y\pm 2)(x^2 y^2 z^2)}{2\sqrt{2}}\left(\cancel{1 + \frac{3\beta x\sqrt{2}}{2}}\right) - \frac{(y\pm 2)^3}{4\sqrt{2}}\left(\cancel{1 + \frac{5\beta x\sqrt{2}}{2}}\right)$$

$$-\frac{\beta x}{\sqrt{2}} - \frac{y}{\sqrt{2}} + \frac{x^2y^2z^2}{2} - \frac{x^2}{4} - \frac{2\beta xy + y^2}{4} + \frac{(\beta x + y)}{2\sqrt{2}}(x^2y^2z^2) - \frac{(\beta x+y)^3}{4\sqrt{2}} - \frac{3z^2(\beta x + y)}{4\sqrt{2}}$$

$$= 2\left\{ 1 - \frac{\beta x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\left(1 + \frac{\beta x}{\sqrt{2}}\right) + \frac{x^2y^2z^2}{2}\left(1 + \frac{\beta x}{\sqrt{2}}\right) - \frac{y^2+z^2}{4}\left(1 + \frac{3\beta x}{\sqrt{2}}\right)\right.$$

$$\left. + \frac{y(x^2y^2z^2)}{2\sqrt{2}}\left(1 + \cancel{\frac{3\beta x}{\sqrt{2}}}\right) - \frac{y^3 + 3xy^2}{4\sqrt{2}}\left(1 + \cancel{\frac{5\beta x}{\sqrt{2}}}\right)\right\}$$

$$1 - \frac{\beta x}{2\sqrt{2}} - \frac{y}{\sqrt{2}} \qquad + \frac{x^2y^2z^2}{2}\left(1 - \frac{\beta x}{2\sqrt{2}}\right) - \frac{x^2+y^2}{4} + \frac{xy\beta}{4} + \frac{y(x^2y^2z^2)}{2\sqrt{2}}$$

$$-\frac{x^3\beta}{8\sqrt{2}} - \frac{y^3}{4\sqrt{2}} - \frac{3x^2y}{4\sqrt{2}} + \frac{9xy^2\beta}{8\sqrt{2}}$$

$$\sum r_2 \rho_6 \rho_8 = 2\left\{ 2 - \frac{3\beta x}{2\sqrt{2}} - \frac{2y}{\sqrt{2}} - \frac{\beta xy}{4} + x^2y^2z^2 - \frac{x^2 + 2y^2 + z^2}{4} + \frac{y(x^2y^2z^2)}{\sqrt{2}}\right.$$

$$+ \frac{x(x^2y^2z^2)\beta}{4\sqrt{2}} - \cancel{\frac{3\beta xy^2}{2\sqrt{2}}} - \frac{3\beta x z^2}{4\sqrt{2}} - \frac{y^3}{2\sqrt{2}} - \frac{3yz^2}{4\sqrt{2}} - \frac{x^3\beta}{8\sqrt{2}}$$

$$\left. - \frac{3x^2y}{4\sqrt{2}} + \frac{3xy^2\beta}{8\sqrt{2}} \right\}$$

$$\frac{9}{8} - \frac{3}{5} = \frac{9-6}{5} \qquad = 2\left\{ 2 - \frac{3\beta x}{2\sqrt{2}} - \frac{2y}{\sqrt{2}} - \frac{\beta xy}{4} + \frac{3x^2 + 2y^2 + 3z^2}{4} + \frac{yx^2}{4\sqrt{2}} + \frac{x^3}{2\sqrt{2}} + \frac{yz^2}{4\sqrt{2}}\right.$$

$$\left. + \frac{3x^3\beta}{8\sqrt{2}} + \frac{xy^2\beta}{8\sqrt{2}} - \frac{xz^2\beta}{[illegible]\sqrt{2}} \right\}$$

$\delta \quad \dots - \beta$

$$r_1 = 1 - \frac{\beta}{2} - \frac{x}{\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y}{\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{x^2+y^2-r^2}{2}\left(1+\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1-\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1+\frac{3\beta}{2}\right) + \frac{xy}{2}\left(1+\frac{\beta}{2}\right)$$
$$+ \frac{x(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{3\beta}{2}\right) - \frac{x^3}{4\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y^3}{4\sqrt2}\left(1+\frac{5\beta}{2}\right) + \frac{3x^2y}{4\sqrt2}\left(1+\frac{\beta}{2}\right) -$$
$$- \frac{3xy^2}{4\sqrt2}\left(1+\frac{3\beta}{2}\right)$$

$$r_2 = 1 + \frac{\beta}{2} - \frac{x}{\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y}{\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{x^2+y^2-r^2}{2}\left(1-\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\beta}{2}\right) - \frac{xy}{2}\left(1-\frac{\beta}{2}\right)$$
$$+ \frac{x(x^2+y^2-r^2)}{2\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y(x^2+y^2-r^2)}{2\sqrt2}\left(1-\frac{3\beta}{2}\right) - \frac{x^3}{4\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y^3}{4\sqrt2}\left(1-\frac{5\beta}{2}\right) - \frac{3x^2y}{4\sqrt2}\left(1-\frac{\beta}{2}\right)$$
$$- \frac{3xy^2}{4\sqrt2}\left(1-\frac{3\beta}{2}\right)$$

$$X_y = -\frac{\partial}{\partial x}\left\{\sum_{2,9,6,8}\Phi + \varphi\,\Sigma(r-l) + \frac{\varphi'}{2}\,\Sigma(r-l)^2 + \frac{\varphi''}{2.3}\,\Sigma(r-l)^3\right\}$$

$$r_9 = 1 - \frac{\beta}{2} + \frac{x}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{y}{\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{x^2+y^2-r^2}{2}\left(1+\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1-\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1+\frac{3\beta}{2}\right) + \frac{xy}{2}\left(1+\frac{\beta}{2}\right)$$
$$- \frac{x(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{y(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{3\beta}{2}\right) + \frac{x^3}{4\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{y^3}{4\sqrt2}\left(1+\frac{5\beta}{2}\right) - \frac{3x^2y}{4\sqrt2}\left(1+\frac{\beta}{2}\right) +$$
$$+ \frac{3xy^2}{4\sqrt2}\left(1+\frac{3\beta}{2}\right)$$

$$r_{29} = 2\left\{1 - \frac{x\beta}{2\sqrt2} - \frac{y}{\sqrt2} + \frac{x^2+y^2-r^2}{2} - \frac{x^2}{4} - \frac{y^2}{4} + \frac{xy\beta}{4} - \frac{x(x^2+y^2-r^2)\beta}{4\sqrt2} + \right.$$
$$\left. + \frac{y(x^2+y^2-r^2)}{2\sqrt2} - \frac{x^3\beta}{8\sqrt2} - \frac{y^3}{4\sqrt2} - \frac{3x^2y}{4\sqrt2} + \frac{9xy^2\beta}{8\sqrt2}\right\}$$

$$\Sigma r_{1,2,9,10} = 4\left\{1 + \frac{xy\,3\beta}{4} + \frac{x^2+y^2-r^2}{2} - \frac{x^2+y^2}{4}\right\} = 2\left\{\cancel{1 + \frac{xy}{2}\left(1+\frac{\beta}{2}\right) + \frac{3x^2+y^2+2r^2}{4}}\right\}$$

Analyse $\quad \operatorname{tg}\varphi = \beta \quad x = x_0 + y\beta$

1). $x = \frac{1}{\sqrt2}(1-\beta) =$ $\quad y = -\frac{1}{\sqrt2}$ $\quad z = 0$ $\quad \boxed{r_0 = 1-\frac{\beta}{\sqrt2}}$

9). $x = -\frac{1-\beta}{\sqrt2}$ $\quad y = \frac{1}{\sqrt2}$ $\quad z = 0$ $\quad \boxed{r_0 = 1-\frac{\beta}{\sqrt2}}$

2). $x = \frac{1+\beta}{\sqrt2}$ $\quad y = \frac{1}{\sqrt2}$ $\quad z = 0$ $\quad \boxed{r_0 = 1+\frac{\beta}{\sqrt2}}$

10). $x = -\frac{1+\beta}{\sqrt2}$ $\quad y = -\frac{1}{\sqrt2}$ $\quad z = 0$ $\quad \boxed{r_0 = 1+\frac{\beta}{\sqrt2}}$

6). $x = \frac{\beta}{\sqrt2}$ $\quad y = \frac{1}{\sqrt2}$ $\quad z = \frac{1}{\sqrt2}$

8). $x = \frac{\beta}{\sqrt2}$ $\quad y = \frac{1}{\sqrt2}$ $\quad z = -\frac{1}{\sqrt2}$

5). $x = -\frac{\beta}{\sqrt2}$ $\quad y = -\frac{1}{\sqrt2}$ $\quad z = \frac{1}{\sqrt2}$

7). $x = -\frac{\beta}{\sqrt2}$ $\quad y = -\frac{1}{\sqrt2}$ $\quad z = -\frac{1}{\sqrt2}$

$\sqrt{(-\frac{\beta}{\sqrt2}-x)^2 + (-\frac{1}{\sqrt2}-y)^2 + (\frac{1}{\sqrt2}-z)^2}$

3). $x = \frac{1}{\sqrt2}$ $\quad y = 0$ $\quad z = -\frac{1}{\sqrt2}$

4). $x = \frac{1}{\sqrt2}$ $\quad y = 0$ $\quad z = \frac{1}{\sqrt2}$

11). $x = -\frac{1}{\sqrt2}$ $\quad y = 0$ $\quad z = -\frac{1}{\sqrt2}$

12). $x = -\frac{1}{\sqrt2}$ $\quad y = 0$ $\quad z = \frac{1}{\sqrt2}$

$r_1^2 = (1-\beta) - (1-\beta)x\sqrt2 + y\sqrt2 + x^2y^2z^2$

$r_{10}^2 = (1+\beta) + (1+\beta)x\sqrt2 + y\sqrt2 +$

$r_9^2 = 1-\beta + (1-\beta)x\sqrt2 - y\sqrt2 +$ $\quad \begin{matrix} \beta & \cdots & -\beta \\ x & & -x \\ y & & -y \end{matrix}$

$r_2^2 = 1+\beta - (1+\beta)x\sqrt2 - y\sqrt2 +$

$r_3^2 = 1 - x\sqrt2 + \cancel{z\sqrt2} + x^2y^2z^2$

$r_4^2 = 1 - x\sqrt2 - z\sqrt2 + \cdots$

$+$

$+$

$r_5^2 = 1 + \beta x\sqrt2 + y\sqrt2 - z\sqrt2 + x^2y^2z^2$

$r_7^2 = 1 + \beta x\sqrt2 + y\sqrt2 + z\sqrt2 + \cdots$

$r_6^2 = 1 - \beta x\sqrt2 - y\sqrt2 - z\sqrt2 + \cdots \quad \cdots 2$ $\quad \begin{matrix} \beta = 0 & \\ x & \beta x \\ y & y+z \end{matrix}$

$r_8^2 = 1 - \beta x\sqrt2 - y\sqrt2 + z\sqrt2 + \cdots$

$$\mathfrak{U} = A + B(x^2+y^2+z^2)$$

$$\frac{\int \alpha x^2 e^{-\lambda x^2}\,dx}{\int e^{-\lambda x^2}\,dx} = \frac{1}{2\lambda}$$

$$\overline{\mathfrak{U}} = A + \iiint \frac{B(x^2+y^2+z^2)\, e^{-B(x^2+y^2+z^2)}\,d\tau}{\iiint e^{-B(x^2+y^2+z^2)}\,d\tau} =$$

$$e^{-\frac{h}{l^2}\left\{x^2\left[\varphi(4-3\delta)+\varphi l(2+3\delta)+\varphi'' l^2\delta\right]\right.}$$

$$\cancel{B = x^2\left[\left(\frac{4\varphi}{l}+2\varphi'\right) = 3\right]} \qquad e^{-\frac{h(y^2+z^2)}{l^2}\left[\varphi\left(4-\frac{\delta}{2}\right)+\varphi l\left(2+\frac{\delta}{2}\right)+\varphi'' l^2\frac{\delta}{2}\right]}$$

[illegible]

$$\overline{\mathfrak{U}} = 12\Phi(l) + 4\varphi l\delta + \underbrace{\frac{3}{2h}}_{=}$$

[illegible] $1+\delta$:

$$\overline{\mathfrak{U}} = n\left\{6\left[\Phi(l) + l\varphi(l)\cdot\delta\right] + \frac{3}{2h}\right\} + \Pi(1+3\delta)$$

[illegible] φ [illegible]

$$\rho\, d\Pi\, c = \frac{3n}{h} + \left[6 n l\varphi + 3\Pi\right]\delta \qquad +\Pi = \left(\frac{4\varphi}{\sqrt{2}} - \frac{M}{2\sqrt{2}h}\right)\frac{1}{l^2}$$

$$n = \frac{1}{\sqrt{2}l^3}$$

$$\rho c = 3n + \left(6 n l\varphi\, \frac{12\varphi}{\sqrt{2}\,l^2} + \frac{3M}{2\sqrt{2}\,l^2}T\right)\frac{\delta}{dT}$$

$$= 3n - \left[\left(\frac{12}{\sqrt{2}} - 6\right) - \frac{3MT}{2\sqrt{2}}\right]\frac{1}{l^2}\,\frac{\delta}{dT}$$

~~4 Punkte auf einen Cubus von $(2\sqrt{2})^3 = 16\sqrt{2}$~~

~~1 Punkt auf : $4\sqrt{2}\, l^3$~~

~~$n = \frac{1}{4\sqrt{2}\, l^3}$~~

Schmelzpunkt = Punkt wo [illegible] = 0

$$\mathfrak{U} = n\left\{6\left[\Phi(l) + \delta l\varphi + \frac{\delta^2}{2} l^2\varphi'\right] + \frac{3}{2h}\right.$$

$$c = \frac{3n}{h} + \left[6 n l\varphi + 6\delta l^2 \varphi'\right]$$

$$\frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)+\frac{3\delta x^2}{4}+\frac{3y^2\delta}{4}-\frac{3x^2y\delta}{2\sqrt2}+\frac{3y(x^2+y^2+z^2)\delta}{2\sqrt2}-\frac{3x^2y\delta}{4\sqrt2}-\frac{3y^3\delta}{4\sqrt2}$$

$$\cancel{2}\frac{y^3}{\cancel{2}\sqrt2}\left(1-\cancel{\tfrac{3}{2}\delta}\right)+\frac{3(x^2+z^2)\delta}{4}+\frac{3y^2\delta}{2}+\frac{3y(x^2+z^2)\delta}{4\sqrt2}-\cancel{\frac{3x^2y\delta}{4\sqrt2}}$$

$$\frac{3y^2}{\sqrt2}+3y\delta+\frac{3(x^2+z^2)y\delta}{4\sqrt2}$$

$$+\overline{V}=2\varphi\left\{-\sqrt2+\frac{\delta}{2\sqrt2}+y\left(1+\frac{\delta}{4}\right)+\frac{x^2+z^2}{4\sqrt2}+\frac{3y^2}{2\sqrt2}-\frac{3(x^2+z^2)y\delta}{8\sqrt2}+\frac{3z^2\delta}{4\sqrt2}\right\}$$

$$+\varphi'\left\{-\frac{\delta}{\sqrt2}+y\left(2-\frac{\delta}{2}\right)-\frac{x^2+z^2}{2\sqrt2}+\frac{3y^2}{\sqrt2}+\frac{3(x^2+z^2)y\delta}{4\sqrt2}+\frac{3z^2\delta}{2\sqrt2}\right\}$$

$$+\varphi''\left\{\frac{y\delta}{2}-\frac{x^2+z^2}{2\sqrt2}-\frac{y^2}{\sqrt2}-\frac{z^2\delta}{2\sqrt2}\right\}$$

$$-l^2(1+\delta)\overline{V}=\frac{4\varphi}{\sqrt2}\left(1-\frac{\delta}{4}\right)+\frac{\varphi' l\delta}{\sqrt2}-\frac{1}{2\sqrt2\,h}\left(M+N'\delta\right)$$

$$\therefore\ -l^2\overline{V}=\frac{4\varphi}{\sqrt2}\left(1-\frac{5\delta}{4}\right)+\frac{\varphi' l\delta}{\sqrt2}-\frac{1}{2\sqrt2\,h}\left(M+(N'-M)\delta\right)$$

Dla równomiernego w wszystkich kierunkach rozszerzenia w stosunku $1+\delta$:

$$+l^2 P=\left\{\frac{4\varphi}{\sqrt2}-\frac{M}{2\sqrt2\,h}+\frac{4\varphi}{\sqrt2}\left(\frac{\delta}{2}-\frac{5\delta}{2}\right)+\frac{4\varphi' l\delta}{\sqrt2}-\frac{\delta}{2\sqrt2\,h}\left(N+2N'-2M\right)\right\}$$

$$=\left\{\frac{4\varphi}{\sqrt2}-\frac{M}{2\sqrt2\,h}+\delta\left\{-\frac{8\varphi}{\sqrt2}+\frac{4\varphi' l}{\sqrt2}-\frac{N+2N'-2M}{2\sqrt2\,k}\right\}\right\}(1-6\delta)\,\Pi\,l^2$$

$$P=-\frac{1}{(1+\delta)l^2}\left\{\frac{4\varphi}{\sqrt2}-\frac{M}{2\sqrt2\,h}+\delta\left[\frac{4\varphi' l}{\sqrt2}-\frac{N'}{2\sqrt2\,h}\right]\right\}-(1-4\delta)\,\Pi$$

①

$$y\left[\sum r_0 - \frac{y y_k}{r_0}\,\cdot\right] - \sum\left\{y_k r_0 - \frac{y\,y_k^2}{r_0} + \frac{x^2+y^2+z^2}{2}\frac{y_k}{r_0} - \frac{y^2}{2}\frac{y_k^3}{r_0^3} - \frac{z^2}{2}\frac{y_k z_k^2}{r_0^3} - \frac{x^2}{2}\frac{y_k x_k^2}{r_0^3}\right\}$$

$$y(4+\delta) + y^2\frac{4}{\sqrt2}\left(1-\frac{\delta}{4}\right) + \frac{4+\delta}{\sqrt2} + y\,2\left(1-\frac{\delta}{4}\right) + \frac{x^2+y^2+z^2}{2\sqrt2}\,4\left(1-\frac{\delta}{4}\right) - \frac{y^2}{2}\frac{4}{2\sqrt2}\left(1-\frac{3\delta}{4}\right)$$

$$-\frac{z^2}{2}\frac{2}{2\sqrt2} - \frac{x^2}{2}\frac{\cancel{2}(1+2\delta)}{\sqrt2\cdot\cancel{2}}\underbrace{\left(1-\frac{3\delta}{2}\right)}_{\left(1+\frac{\delta}{2}\right)}$$

$$= \frac{4+\delta}{\sqrt2} + y\left(6+\frac{\delta}{2}\right) + \frac{x^2}{2\sqrt2}\left(4-\delta-1-\frac{\delta}{2}\right) + \frac{y^2}{2\sqrt2}\left(8-2\delta+4-\delta-2+\frac{3\delta}{2}\right)$$

$$+\frac{z^2}{2\sqrt2}(4-\delta-1)$$

$$= \frac{4+\delta}{\sqrt2} + y\left(6+\frac{\delta}{2}\right) + \frac{x^2}{2\sqrt2}\left(3-\frac{3\delta}{2}\right) + \frac{y^2}{2\sqrt2}\left(10-\frac{3\delta}{2}\right) + \frac{z^2}{2\sqrt2}(3-\delta)$$

$$\frac{\partial}{\partial y}\sum(r_k-l)^2 = \frac{\partial}{\partial y}\left(\sum r_k^2 - 2l\frac{\partial}{\partial y}\sum r_k\right)$$

$$\frac{\partial}{\partial y}\,4\left(1+\delta+y\sqrt2+x^2+y^2+z^2\right) = 4\left(\sqrt2+2y\right).$$

$$-\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{3x^2\delta}{4} + \frac{3y^2\delta}{4} - \frac{3xy\delta}{2} + 3\delta\left\{-\frac{x(x^2+y^2+z^2)}{2\sqrt2} + \frac{x^3}{4\sqrt2} + \right.$$

$$\left. + \frac{xy^2}{4\sqrt2} - \cancel{\frac{x^2y}{2\sqrt2}} + \frac{y(x^2+y^2+z^2)}{2\sqrt2}\right\}$$

$$= -\frac{x^3}{2\sqrt2}\left(1+3\frac{\delta}{2}\right) + \frac{3x^2\delta}{4} + \frac{3y^2\delta}{4} - \frac{3\delta x(x^2+y^2+z^2)}{2\sqrt2} + \frac{3\delta x^3}{4\sqrt2} + \cancel{\frac{3\delta xy^2}{4\sqrt2}}$$

$$= -\frac{x^3}{2\sqrt2}\left[1+\cancel{\frac{3\delta}{2}} - 3\delta + \cancel{\frac{3\delta}{2}}\right] + \frac{3x^2\delta}{4} - \frac{3y^2\delta}{4} - \frac{3\delta x y^2}{4\sqrt2} - \frac{3\delta x z^2}{2\sqrt2}$$

$$-\frac{z^3}{2\sqrt2} \quad \cancel{+\frac{3x^2\delta}{4} - \frac{3y^2\delta}{4} - \frac{3\delta xy^2}{4\sqrt2} - 3\delta} \quad -\frac{3y\delta}{2} - \frac{3xy\delta}{2\sqrt2}$$

$$\cancel{-\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right)} + \frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right) + \frac{3\delta}{2}\Big\{ \frac{x^2}{2}\cancel{(1+\delta)} + \frac{y^2}{2}\cancel{(1-\delta)} \cancel{- xy}$$

$$\cancel{-\frac{x(x^2+y^2)}{\sqrt2}} + \cancel{\frac{x^3}{2\sqrt2}(1+\delta)} + \cancel{\frac{xy^2}{2\sqrt2}(1-\delta)} - \frac{x^2y}{\sqrt2} + y\frac{(x^2+y^2)}{\sqrt2}\cancel{(1-\delta)}$$

$$-\frac{x^2y}{2\sqrt2} - \frac{y^3}{2\sqrt2}\cancel{(1-2\delta)} + \cancel{\frac{xy^2}{\sqrt2}(1-\delta)}\Big\}$$

$2\left(1-\frac{3\delta}{2}\right)+2$
$=4$

$$\frac{3y^2}{2\sqrt2}\left(1-\frac{3\delta}{2}\right) + \frac{3\delta}{2}\left(y - \frac{x^2}{\sqrt2} + \frac{x^2+3y^2+z^2}{\sqrt2} - \frac{x^2}{2\sqrt2} - \frac{3y^2}{2\sqrt2}\right) + \frac{3x^2}{2\sqrt2}\left(1+\frac{\delta}{2}\right)$$

$$\frac{3y^2}{2\sqrt2} - \cancel{\frac{9y^2\delta}{4\sqrt2}} + \frac{3\delta}{2}\left(y - \frac{x^2}{2\sqrt2} + \cancel{\frac{3y^2}{2\sqrt2}} + \frac{z^2}{\sqrt2}\right)$$

$-\frac{1}{4}\ \frac{1}{\sqrt2}$

$$\frac{3y^2}{2\sqrt2} \qquad + 3\frac{\delta}{2}\left(y - \frac{z^2}{2\sqrt2} + \frac{x^2}{\sqrt2}\right)$$

$$+\frac{3(x^2+z^2)}{2\sqrt2}$$

$$2\ \boxed{\frac{3y^2}{\sqrt2} + 3y\delta + \frac{3}{2\sqrt2}(x^2+z^2)\delta}$$

$4y + \frac{4}{\sqrt2}$

$\cancel{x} - \frac{x}{2\sqrt2} + \cancel{y} + \frac{y\delta}{4}$

$$\underbrace{\left(\varphi - \varphi' + \frac{\varphi''}{2}\right)}_{B}\frac{\partial z}{\partial x} + \underbrace{(\varphi' - \varphi'')}_{2C}\, z\frac{\partial z}{\partial x} + \underbrace{\frac{\varphi''}{2}}_{3D}\, z^2\frac{\partial z}{\partial x}$$

$+\frac{1}{\sqrt2} + \frac{\delta}{2\sqrt2} + \cancel{y} + \frac{y\delta}{4}$

$$\varphi\frac{\partial z}{\partial x} - \varphi'\left(\frac{\partial z}{\partial x} - z\frac{\partial z}{\partial x}\right) + \varphi''\left(\frac{1}{2}\frac{\partial z}{\partial x} - z\frac{\partial z}{\partial x} + \frac{1}{2}z^2\frac{\partial z}{\partial x}\right)$$

$-\cancel{4y} - \frac{4}{\sqrt2}$

$$\frac{4-\delta}{2\sqrt2} + \frac{4+\delta}{2\sqrt2} - \frac{4}{\sqrt2} \qquad y+3y-4y \qquad \frac{y\delta}{4} \qquad -\frac{x^2}{4\sqrt2} + \frac{3x^2}{4\sqrt2} \qquad -\frac{3y^2}{2\sqrt2} + \frac{5y^2}{2\sqrt2}$$

$$\frac{x^2}{2\sqrt2} \qquad +\frac{2y^2}{2\sqrt2}$$

$$\frac{3x^2\delta}{8\sqrt2} - \frac{3x^2\delta}{8\sqrt2} \qquad \frac{3y^2\delta}{8\sqrt2} - \frac{3y^2\delta}{8\sqrt2} \qquad \frac{3}{4} - \frac{1}{4} \qquad \frac{z^2\delta}{2\sqrt2}$$

$$\frac{3}{2\sqrt2}\ \frac{1}{4} + \frac{9}{8\sqrt2} - \cancel{\frac{3\delta}{8\sqrt2}}$$

$$E\frac{\partial u}{\partial x} = X_x - \mu(Y_y + Z_z)$$

$$E\frac{\partial v}{\partial y} = Y_y - \mu(X_x + Z_z)$$

$$E\frac{\partial w}{\partial z} = Z_z - \mu(X_x + Y_y)$$

$$X_x = \frac{E\begin{vmatrix} \frac{\partial u}{\partial x} & -\mu & -\mu \\ \frac{\partial v}{\partial y} & 1 & -\mu \\ \frac{\partial w}{\partial z} & -\mu & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{vmatrix}} = E\,\frac{\frac{\partial u}{\partial x}(1-\mu^2) - \frac{\partial v}{\partial y}(-\mu-\mu^2) + \frac{\partial w}{\partial z}(\mu+\mu^2)}{1-\mu^2 + \mu(-\mu-\mu^2) - \mu(\mu+\mu^2)}$$

$$= E\,\frac{\frac{\partial u}{\partial x}(1-\mu) + \mu\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)}{1-\mu-2\mu^2} \qquad = (1+\mu)(1-2\mu)$$

$= 2:1$

$\frac{\mu}{1+\mu}$ $\frac{2(1-\mu)}{1-\mu}$: ... $= 2(1-\mu):1$

$$= E\left[\frac{\partial u}{\partial x}\left(\frac{1-\mu}{1-3\mu}\right) + \frac{\mu}{1-3\mu}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right]$$

$$= E\left[\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\frac{\mu}{1-3\mu} + \frac{\partial u}{\partial x}\left(\frac{1-2\mu}{1-3\mu}\right)\right]$$

$$\frac{X_x}{E} = \frac{\partial u}{\partial x}\,\frac{1-\mu}{(1-2\mu)(1+\mu)} + \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\frac{\mu}{(1-2\mu)(1+\mu)}$$

$1+\mu$	0	$-\mu$	0	$1+\mu$	-2μ
$-(\mu+1)$	$1+\mu$	$-\mu$	$-(\mu+1)$	$1+\mu$	$-\mu$
0	$-(\mu+1)$	1	0	$-(\mu+1)$	

$-(1+\mu)$

$1-\mu-2\mu^2$

$$= \frac{E\mu}{(1+\mu)(1-2\mu)}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \frac{E\mu}{1+\mu}\,\frac{\partial u}{\partial x}$$

$$= \frac{E\mu(1-\mu)2}{(1+\mu)(1-2\mu)}\,\frac{\partial u}{\partial x} + \frac{E\mu}{(1+\mu)(1-2\mu)}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\underset{15740}{\Sigma}\, Y = \Sigma\left(B\,\frac{\partial r}{\partial y} + 2C\,r\,\frac{\partial r}{\partial y} + 3D\,r^2\,\frac{\partial r}{\partial y}\right)$$

$$= B\,\Sigma\,(y-y_0)\left[\frac{1}{r_0} + \ldots\right.$$

$$= B\left\{y\,\Sigma\frac{1}{r_0} + y^2\,\Sigma\frac{y_\kappa}{r_0^3} + y z\,\cancel{\Sigma\frac{z_\kappa}{r_0^3}} + y x\,\cancel{\Sigma\frac{x_\kappa}{r_0^2}} - \Sigma\frac{y_\kappa}{r_0} - y\,\Sigma\frac{y_\kappa^2}{r_0^3} - z\,\cancel{\Sigma\frac{y_\kappa z_\kappa}{r_0^2}}\right.$$

$$\left. - x\,\cancel{\Sigma\frac{y_\kappa x_\kappa}{r_0^2}} + \frac{x^2+y^2+z^2}{2}\,\Sigma\frac{y_\kappa}{r_0^3} - \frac{y^2}{2}\,\Sigma\frac{3y_\kappa^3}{r_0^5} - \frac{z^2}{2}\,\Sigma\frac{3y_\kappa z_\kappa^2}{r_0^5} - \frac{x^2}{2}\,\Sigma\frac{3y_\kappa x_\kappa^2}{r_0^5}\right\}$$

$$+ 8Cy - 2C\,\Sigma y_\kappa + D\ldots$$

$$-l^2\overline{\underline{X}} = \frac{4\varphi}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi' l\delta}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,k}(M + N\delta)$$

$$-l^2\overline{X}_0 = \frac{4\varphi}{\sqrt{2}} - \frac{M}{2\sqrt{2}\,k}$$

w tem l oznacza m.p. odstępy dla $T=0$, $\mu=0$

w takim razie tutaj ostrzenie przy temp. T w takich odstępach utrzymane

Dwie drogi: albo ograniczamy się na trzech członach siły: φ, φ' φ''.

~~w takim razie wezmiemy~~

Wtedy siła $X_0 = \alpha T \dfrac{E}{1-2\mu}$ (α = lin. ekspans. coeff.)

$\cancel{X = \frac{E}{3(1-2\mu)}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)}$

$\cancel{X = E\left[\frac{\partial u}{\partial x} - \mu\,\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right]}$

$$\Sigma \frac{1}{r_0} = 2\left(1-\frac{\delta}{2}\right)+2 = 4-\delta$$

$$\Sigma \frac{1}{r_0^3} = 2\left(1-\frac{3\delta}{2}\right)+2 = 4-3\delta$$

$$= B\Big\{ y\left(4-\delta\right) + y^2\frac{(4-3\delta)}{\sqrt{2}} + \frac{(4-\delta)}{\sqrt{2}} - y.2\left(1-\frac{3\delta}{4}\right) + \frac{x^2+y^2+z^2}{2}\frac{(4-3\delta)}{\sqrt{2}}$$

$$+ \frac{3y^2}{2}\frac{(4-5\delta)}{2\sqrt{2}} + \frac{3z^2}{2}\frac{2}{2\sqrt{2}} + \frac{3x^2}{2\sqrt{2}.2}(1+2\delta)\underbrace{\left(1-\frac{5\delta}{2}\right)}_{1+\frac{3\delta}{2}}$$

$$+ 8Cy + 2C\frac{4}{\sqrt{2}} + D \ldots$$

$-6+\frac{3\delta}{2}$

$$= B\left\{\frac{4-\delta}{\sqrt{2}} + y\left(2+\frac{\delta}{2}\right) + \frac{y^2}{2\sqrt{2}}\left(-4+3\delta - 8 + 6\delta + 6 - \frac{15\delta}{2}\right) + \frac{x^2}{\sqrt{2}}\left(-4+3\delta+\frac{3}{2}-\frac{3}{4}\delta\right)\right.$$

$-\frac{5}{2}+\frac{9\delta}{4}$

$-1+\frac{3}{2}\delta$

$$+\frac{z^2}{2\sqrt{2}}\left(-4+3\delta+3\right)$$

$(-1+3\delta)$

$$\overline{X} = -\frac{1}{2\sqrt{2}\,h\,l}\left\{ \frac{3\varphi\left(1-\frac{3\delta}{2}\right) - 3\varphi' l\left(1-\frac{3\delta}{2}\right) - \varphi'' l^2 (1+3\delta)}{\varphi(4-3\delta) + \varphi' l (2+3\delta) + \varphi'' l^2 \delta} + \right.$$

$$\left. + \frac{\varphi\left(1+5\frac{\delta}{2}\right) - \varphi' l\left(1+5\frac{\delta}{2}\right) - \varphi'' l^2}{\varphi\left(4-\frac{\delta}{2}\right) + \varphi' l\left(2+\frac{\delta}{2}\right) + \varphi'' l^2 \frac{\delta}{2}} \right\} + 4\varphi\left\{\frac{1}{\sqrt{2}} + \frac{\delta}{2\sqrt{2}}\right\} + \frac{2\varphi' l \delta}{\sqrt{2}}$$

$$\{\ \} = \frac{3\varphi - 3\varphi' l - \varphi'' l^2 \cancel{+} \frac{9\varphi\delta}{2} + 9\frac{\varphi' l\delta}{2} - 3\varphi'' l^2 \delta}{(4\varphi + 2l\varphi')\left(1 + \frac{-3\varphi + 3\varphi' l + \varphi'' l^2}{4\varphi + 2l\varphi} \delta\right)} + \ldots$$

$$\frac{A+\alpha}{B+\beta} = \frac{A\left(1+\frac{\alpha}{A}\right)}{D\left(1+\frac{\beta}{D}\right)} = \frac{A}{B}\left(1+\frac{\alpha}{A}\right)\left(1-\frac{\beta}{B}\right) = \frac{A}{D}\left(1+\frac{\alpha}{A}-\frac{\beta}{B}\right)$$

$$= \frac{A}{B} + \cancel{\frac{\alpha}{B}} \frac{\alpha B - \beta A}{B^2}$$

$$\{\ \} = \frac{3\varphi - 3\varphi' l - \varphi'' l^2 + \varphi - \varphi' l - \varphi'' l^2}{2(2\varphi + l\varphi')} + \delta(4\varphi + 2l\varphi')\left[-\frac{9\varphi}{2} + \frac{9\varphi' l}{2} - 3\varphi'' l^2 + \frac{5\varphi}{2} - \frac{5\varphi' l}{2}\right]$$

$$\frac{-(-3\varphi + 3\varphi' l + \varphi'' l^2)(3\varphi - 3\varphi' l - \varphi'' l^2) - \left(-\frac{\varphi}{2} + \frac{\varphi' l}{2} + \frac{\varphi'' l^2}{2}\right)(\varphi - \varphi' l - \varphi'' l^2)}{(4\varphi + 2l\varphi')^2}$$

$$= \frac{4\varphi - 4\varphi' l - 2\varphi'' l^2}{4\varphi + 2l\varphi'} + \delta \frac{(4\varphi + 2l\varphi')(-2\varphi + 2\varphi' l - 3\varphi'' l^2) + (3\varphi - 3\varphi' l - \varphi'' l^2)^2 + \frac{1}{2}(\varphi - \varphi' l - \varphi'' l^2)^2}{(4\varphi + 2l\varphi')^2}$$

$$= \overbrace{\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l}}^{M} + \delta \overbrace{\frac{3\varphi^2 - 30\varphi\varphi' + 27\varphi'^2 - 38\varphi\varphi'' + 3\varphi''^2 + 2\varphi'\varphi''}{8(2\varphi + \varphi' l)^2}}^{N}$$

$$6x^2\delta + 3(y^2+z^2)\delta + 12 - 36\left(1-\frac{\delta}{3}\right) - 3x^2(4-3\delta) - 3(y^2+z^2)\left(4-\frac{\delta}{2}\right)$$

$$+\,36\left(1+\frac{2\delta}{3}\right) \cancel{+36(y^2+z^2)}$$

$$= 12(1+\delta) + x^2(24+15\delta) + (y^2+z^2)\left(24+\frac{9\delta}{2}\right)$$

$$-2 + 6\left(1+\frac{\delta}{3}\right) + x^2\left(2-\frac{3\delta}{2}\right) + (y^2+z^2)\left(2-\frac{\delta}{4}\right)$$

$$-6\left(1+\frac{2\delta}{3}\right) - 6x^2 - 6(y^2+z^2)$$

$$+2(1+\delta) + 3(x^2+y^2+z^2)\left(1+\frac{\delta}{3}\right) + x^2\left(1+\frac{3\delta}{2}\right) + (y^2+z^2)\left(1-\frac{\delta}{4}\right)$$

$$x^2\left(\cancel{2-6+3+1} - \frac{3\delta}{2} + \delta + \frac{3\delta}{2}\right) + (y^2+z^2)\left(2-\frac{\delta}{4} - 6 + 3 + \delta + 1 - \frac{\delta}{4}\right)$$

$$+\overline{X} = 4\varphi\left[\frac{1}{\sqrt{2}} + \frac{\delta}{2\sqrt{2}}\right] + \frac{2\varphi l\delta}{\sqrt{2}} - \iiint$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\int x^2 e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\frac{\int x^2 e^{-\alpha x^2}dx}{\int e^{-\alpha x^2}dx} = \frac{1}{2\alpha} \qquad \frac{\int (y^2+z^2)\, e^{-\alpha(y^2+z^2)}dy\,dz}{\int e^{-\alpha(y^2+z^2)}dy\,dz} = \frac{1}{\alpha}$$

$$-8\varphi^2 - 4\varphi\varphi' + 8\varphi\varphi' + 4\varphi'^2 - 12\varphi\varphi'' - 6\varphi'\varphi''$$

$$+9\varphi^2 - 9\varphi\varphi' - 9\varphi\varphi' + 9\varphi'^2 - 3\varphi\varphi'' + 3\varphi'\varphi'' - 3\varphi\varphi'' + 3\varphi'\varphi'' + \varphi''^2$$

$$2(\varphi^2 - 14\varphi\varphi' + 13\varphi'^2 - 18\varphi\varphi'' + \varphi''^2)$$

$$+\varphi^2 - 2\varphi\varphi' \quad + \varphi'^2 \quad -2\varphi\varphi'' \quad + \varphi''^2 \quad +2\varphi'\varphi''$$

$\varphi' > 0 \quad \varphi'' \sim 0$

$$4\varphi'\left\{-\frac{1+\frac{\delta}{2}}{\sqrt{2}}+\frac{x}{2}\left(1-\frac{3\delta}{2}\right)+\frac{5x^2}{8\sqrt{2}}\left(1-\frac{23\delta}{10}\right)+\frac{y^2+z^2}{8\sqrt{2}}\left(1+\frac{5\delta}{2}\right)+\frac{1+\delta}{\sqrt{2}}\cancel{+x}\right\}$$

$$=-2\varphi\left\{+\frac{\delta}{4\sqrt{2}}-\frac{x}{4}\left(1+\frac{3\delta}{2}\right)+\frac{5x^2}{4\sqrt{2}}\left(1-\frac{23\delta}{10}\right)+\frac{y^2+z^2}{4\sqrt{2}}-\right\}$$

$$2\varphi''\left\{-\frac{1+\frac{\delta}{2}}{\sqrt{2}}+\frac{x}{2}\left(1-\frac{3\delta}{2}\right)+\frac{5x^2}{8\sqrt{2}}\left(1-\frac{23\delta}{10}\right)+\frac{x^2+y^2+z^2}{8\sqrt{2}}\left(1+\frac{5\delta}{2}\right)-\cancel{2}x+\frac{\cancel{2}(1+\delta)}{\sqrt{2}}\right.$$

$$\left.\cancel{+}\frac{1+\frac{3\delta}{2}}{\sqrt{2}}+\frac{3x}{2}\left(1+\frac{5\delta}{6}\right)+\frac{x^2}{8\sqrt{2}}\left(-7-\frac{\delta}{2}\right)+\frac{y^2+z^2}{8\sqrt{2}}\left(3+\frac{5\delta}{2}\right)\right\}$$

$$\cancel{\frac{1+\frac{3\delta}{2}}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\left(1-\frac{\delta}{2}+1+\delta\right.}\cdots$$

$$x\left(\frac{1}{2}-\frac{3\delta}{4}-1+\frac{3}{2}+\frac{5\delta}{4}\right)$$

$$\cancel{\frac{1}{\sqrt{2}}\left(-1-\frac{\delta}{2}+2+2\delta-1-\frac{3\delta}{2}\right.}\qquad \cancel{x(\ \ )}\qquad \frac{x\delta}{2}$$

$$\frac{x^2}{8\sqrt{2}}\left(5-7-\frac{23}{2}\delta-\frac{\delta}{2}+1+\cancel{\frac{5\delta}{2}}-3-\cancel{\frac{5}{2}}\right)$$

$$\frac{y^2+z^2}{8\sqrt{2}}\left(1+\frac{5\delta}{2}-3-\frac{5\delta}{2}\right)=\frac{y^2+z^2}{8\sqrt{2}}(-2)$$

$$4(x^2+y^2+z^2)+\delta(6x^2+y^2+z^2)-12+24\left(1+\frac{\delta}{3}\right)+2x^2(4-3\delta)+\cancel{4}(y^2+z^2)\left(8-\frac{\delta}{4}\right)$$

$$=12+8\delta+\cancel{4}(12(x^2+y^2+z^2)\qquad \cancel{\ \ }$$

$$3x^2\delta+3(y^2+z^2)\delta+12-12\delta\cancel{+12(x^2+y^2+z^2)}+9x^2\delta+\frac{3}{2}(y^2+z^2)\delta$$

$$+\cancel{4(x^2+y^2+z^2)}+9x^2\delta+\cancel{4}(y^2+z^2)$$

$$=\cancel{12(1-\delta)+33x^2\delta}$$

$$=12(1-\delta)-8(x^2+y^2+z^2)+21x^2\delta+\frac{11}{2}(y^2+z^2)\delta$$

$$\frac{\partial r}{\partial x} = 2\left(1+\frac{\delta}{2}\right)\left[-\frac{1}{\sqrt{2}} + \frac{x}{1+\delta} - \frac{x}{2} + \cancel{\frac{y}{2(1+\delta)}} + \frac{3x^2}{2\sqrt{2}\,(1+\delta)} + \frac{y^2+z^2}{2\sqrt{2}\,(1+\delta)}\right.$$

$$\left. - \cancel{\frac{2xy}{2\sqrt{2}\,(1+\delta)^2}} + \frac{2\sqrt{2}}{16}\left(-3x^2 + \cancel{\frac{6xy}{1+\delta}} - \frac{3y^2}{(1+\delta)^2}\right)\right.$$

$$\sum_{1234} \frac{\partial r}{\partial x} = 4\left(1+\frac{\delta}{2}\right)\left[-\frac{1}{\sqrt{2}} + \frac{x}{1+\delta} - \frac{x}{2} + \frac{3x^2}{2\sqrt{2}\,(1+\delta)} - \frac{3x^2}{4\sqrt{2}} + \frac{y^2+z^2}{2\sqrt{2}\,(1+\delta)} - \frac{3}{8\sqrt{2}}\frac{(z^2+y^2)}{(1+\delta)^2}\right]$$

$$-\frac{1}{\sqrt{2}} + \frac{x(2-1-\delta)}{2(1+\delta)} + \frac{3x^2\,(2-(1+\delta))}{4\sqrt{2}\,(1+\delta)} + \frac{y^2+z^2}{8\sqrt{2}}\left(\frac{4}{1+\delta} - \frac{3}{(1+\delta)^2}\right)\Big]$$

$$-\frac{1}{\sqrt{2}} + \frac{x}{2}(1-2\delta) + \frac{3x^2(1-2\delta)}{4\sqrt{2}} + \frac{y^2+z^2}{8\sqrt{2}}\,(4-4\delta-3+6\delta)$$

$$\overset{\shortparallel}{1+2\delta}$$

$$\sum \frac{\partial r}{\partial x} = 4\left\{-\frac{1+\frac{\delta}{2}}{\sqrt{2}} + \frac{x}{2}\left(1-\frac{3\delta}{2}\right) + \frac{3x^2}{4\sqrt{2}}\left(1-\frac{3\delta}{2}\right) + \frac{y^2+z^2}{8\sqrt{2}}\left(1+\frac{5\delta}{2}\right)\right\}$$

$$\frac{\partial r}{\partial x} = \frac{\frac{1+\delta}{\sqrt{2}} - x}{\sqrt{\ldots}}$$

$$\sum X = B \sum \frac{\partial r}{\partial x} + C \sum 2r\frac{\partial r}{\partial x} + D \sum 3r^2\frac{\partial r}{\partial x}$$

$$\overset{\shortparallel}{\frac{\varphi''}{2}} \qquad = -\frac{\varphi''}{2} \qquad \frac{\varphi''}{2\cdot 3}$$

$$= \varphi''\left\{\frac{\sum \frac{\partial r}{\partial x}}{2} - \frac{\sum 2r\frac{\partial r}{\partial x}}{2} + \frac{\sum 3r^2\frac{\partial r}{\partial x}}{2\cdot 3}\right.$$

$$2_B \quad - \quad 4_C \quad + \quad 2_D$$

$-1-\frac{1}{2}+\frac{1}{4}$

$\cancel{\sum x_k = 4\frac{(1+\delta)}{\sqrt{2}}}$

$$12D\left\{-\frac{\left(1+\frac{3\delta}{2}\right)}{\sqrt{2}} + x\left[\frac{\left(1+\frac{3\delta}{2}\right)}{2}+1+\frac{\delta}{2}\right] - x^2\frac{\left(1+\frac{\delta}{2}\right)}{\sqrt{2}} - \frac{x^2+y^2+z^2}{2\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \right.$$

$$\left. + \frac{x^2}{4\sqrt{2}}\left(1+\frac{3\delta}{2}\right) + \frac{(x^2+y^2+z^2)}{8\sqrt{2}}\left(1-\frac{\delta}{2}\right)\right\}$$

$$= 12D\left\{-\frac{1+\frac{3\delta}{2}}{\sqrt{2}} + \left(\frac{3}{2}+\frac{5\delta}{4}\right)x + (x^2+y^2+z^2)\left[\frac{1}{8\sqrt{2}}\left(1-\frac{\delta}{2}\right) - \frac{1}{2\sqrt{2}}\left(1+\frac{\delta}{2}\right)\right] + \right.$$

$$\left. + \frac{x^2}{8\sqrt{2}}\left[\underbrace{2\left(1+\frac{3\delta}{2}\right) - \left(1-\frac{\delta}{2}\right)}_{\left(-7-\frac{\delta}{2}\right)} - 8\left(1+\frac{\delta}{2}\right)\right]\right.$$

$$\frac{1}{8\sqrt{2}}\left(1-\frac{\delta}{2}-4-2\delta\right)$$

$$\left(-3-\frac{5\delta}{2}\right)$$

$$= 12D\left\{-\frac{1+\frac{3\delta}{2}}{\sqrt{2}} + \frac{3x}{2}\left(1+\frac{5\delta}{6}\right) + \frac{x^2}{8\sqrt{2}}\left(-7-\frac{\delta}{2}\right) - \frac{x^2+y^2+z^2}{8\sqrt{2}}\left(3+\frac{5\delta}{2}\right)\right\}$$

$$+8C\left\{x-\frac{1+\delta}{\sqrt{2}}\right\}$$

$$+4B\left\{-\frac{1+\frac{\delta}{2}}{\sqrt{2}} + \frac{x}{2}\left(1-\frac{3}{2}\delta\right) + \frac{5x^2}{8\sqrt{2}}\left(1-\frac{22\delta}{10}\right) + \frac{x^2+y^2+z^2}{8\sqrt{2}}\left(1+\frac{5\delta}{2}\right)\right\}$$

stimmt

$$-\frac{1}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + x\left(1-\frac{\delta}{2}-\frac{1}{2}-\frac{\delta}{4}\right) + \frac{3x^2}{2\sqrt{2}}\left(1-\frac{\delta}{2}\right) - \frac{3x^2}{4\sqrt{2}}\left(1+\frac{\delta}{2}\right)$$

$$\frac{1}{2}-\frac{3\delta}{4} \qquad -\frac{3\delta x^2}{4\sqrt{2}} - \frac{3\delta x^2}{8\sqrt{2}}$$

$$-\frac{25}{2}+\frac{5}{2} = \frac{30}{2}$$

$$\frac{15}{8\sqrt{2}}$$

$$\frac{1}{2\sqrt{2}}\left(1-\frac{\delta}{2}\right) - \frac{3}{8\sqrt{2}}\left(1-\frac{3\delta}{2}\right) = \frac{1}{8\sqrt{2}}\left(4-2\delta-3+9\frac{\delta}{2}\right)$$

$$1+\frac{5\delta}{2}$$

$$\frac{1}{4\sqrt{2}}\left(1-\frac{7\delta}{2}\right) + \frac{3}{8\sqrt{2}}\left(1-\frac{3\delta}{2}\right) = \cancel{\left(1-\frac{3\delta}{2}\right)\left(\frac{3}{8}+\frac{1}{4}\right)}$$

$$-\frac{23}{10\cdot 2}\cancel{5}+\frac{5}{2} =$$

$$= \frac{1}{8\sqrt{2}}\left(2-7\delta+3-9\frac{\delta}{2}\right) = 5-\frac{23}{2}$$

$$B\left\{4x\left(1-\frac{\delta}{2}\right) + 4x^2\left(1-\frac{3}{2}\delta\right)\frac{(1+\delta)}{\sqrt{2}} - \frac{4(1+\delta)}{\sqrt{2}}\left(1-\frac{\delta}{2}\right) - x\,\frac{4\overbrace{(1+2\delta)}^{1+\frac{\delta}{2}}}{2}\left(1-\frac{3\delta}{2}\right)\right.$$

$$+\frac{x^2+y^2+z^2}{2}\,\frac{4(1+\delta)}{\sqrt{2}}\left(1-\frac{3\delta}{2}\right) - \frac{3x^2}{2}\,\frac{4(1+3\delta)}{2\sqrt{2}}\left(1-\frac{5\delta}{2}\right)$$

$$\left.-\frac{3(y^2+z^2)}{2}\frac{1+\delta}{\sqrt{2}}\left(1-\frac{5\delta}{2}\right)\right\} + 8C\left\{x - \frac{(1+\delta)}{\sqrt{2}}\right\}$$

$$\cancel{= 4B\left\{x\left(1-\frac{\delta}{2}-\frac{1}{2}-\frac{\delta}{4}\right) - \frac{\left(1+\frac{\delta}{2}\right)}{\sqrt{2}} + \frac{x^2}{\sqrt{2}}\left(1-\frac{3}{4}\left(1+\frac{\delta}{2}\right)\right)\right.}$$

$$= 4B\left\{\begin{array}{l} -\frac{\left(1+\frac{\delta}{2}\right)}{\sqrt{2}} + x\frac{(1-\frac{3}{2}\delta)}{2} + \frac{x^2}{\sqrt{2}}\left(\frac{1}{4}-\frac{7\delta}{8}\right) - \\ -\frac{3(y^2+z^2)}{8\sqrt{2}}\left(1-\frac{3\delta}{2}\right) + \frac{x^2+y^2+z^2}{2\sqrt{2}}\left(1-\frac{\delta}{2}\right)\Big\} + 8C\left\{x-\frac{1+\delta}{\sqrt{2}}\right\}\end{array}\right\}$$

3 D) $\sum r(x-x_k) = x\sum\Big\{r_0 - \cancel{\frac{x x_k + y y_k + z z_k}{r_0}} + \cancel{\frac{x^2+y^2+z^2}{2}\frac{1}{r_0}}$

$\cancel{-\frac{x^2}{2}\frac{x_k^3}{r_0^3} - \frac{y^2}{2}\frac{y_k^2}{r_0^3} - \frac{z^2}{2}\frac{z_k^2}{r_0^3} - \frac{xy\, x_k y_k + x z\, x_k z_k + y z\, y_k z_k}{r_0^3}}\Big\}$

$$-\sum\left\{x_k r_0 - \frac{x x_k^2}{r_0} + \frac{x^2+y^2+z^2}{2}\frac{x_k}{r_0} - \frac{x^2}{2}\frac{x_k^3}{r_0^3} - \frac{y^2}{2}\frac{x_k y_k^2}{r_0^3} - \frac{z^2}{2}\frac{x_k z_k^2}{r_0^3}\right\}$$

$\cancel{-xy\,x_k y_k r}$

$$= x\left\{4\left(1+\frac{\delta}{2}\right) - x\left(1-\frac{\delta}{2}\right)\frac{4(1+\delta)}{\sqrt{2}}\right\} - \frac{4\left(1+\frac{\delta}{2}\right)(1+\delta)}{\sqrt{2}} + x\,\frac{4(1+2\delta)}{2}\left(1-\frac{\delta}{2}\right)$$

$$-\frac{x^2+y^2+z^2}{2}\,\frac{4(1+\delta)}{\sqrt{2}}\left(1-\frac{\delta}{2}\right) + \frac{x^2}{2}\,\frac{4(1+3\delta)}{2\sqrt{2}}\left(1-\frac{3\delta}{2}\right) + \frac{(y^2+z^2)}{2}\frac{1+\delta}{\sqrt{2}}\left(1-\frac{3\delta}{2}\right)$$

$$\sum \frac{x_k^2}{r_0^5} = 4(1+2\delta)\left(1-\frac{5\delta}{2}\right) = 4\left(1-\frac{\delta}{2}\right)$$

$$\sum \frac{y_k^2}{r_0^5} = 2\left(1-\frac{5\delta}{2}\right)+2 = 4\left(1-\frac{5}{4}\delta\right)$$

$$r^2 = x_k^2+y_k^2+z_k^2 + x^2+y^2+z^2 - 2(x x_k + y y_k + z z_k)$$

$$r\frac{\partial r}{\partial x} = 2(x-x_k)$$

$$\sum r\frac{\partial r}{\partial x} = 24x$$

$$\sum r^2\frac{\partial r}{\partial x} = \sum r^2\frac{x-x_k}{r} = \sum r(x-x_k) = x\sum r_k - \sum r_k x_k$$

$$\sum r_k x_k = -x\sum\frac{x_k^2}{r_0}$$

$$\sum r^2\frac{\partial r}{\partial x} = 12x\left(1+\frac{\delta}{3}\right) + \cancel{x^3(4-3\delta)} + \cancel{x(y^2+z^2)(4-\delta)} - x.4\underbrace{(1+2\delta)\left(1-\frac{\delta}{2}\right)}_{1+\frac{3\delta}{2}}$$

$$= x\left[12+4\delta-4-6\delta\right] = x\left[8-2\delta\right]$$

$$\sum X = x\left\{B(1-\delta)8. + 2C.24 + 3D(8-2\delta)\right\}$$

$$\sum_{1234} X = B\sum(x-x_k)\left[\frac{1}{r_0} + \frac{x x_k + y y_k + z z_k}{r_0^2} + \cdots\right.$$

$$= B\left\{x\sum\frac{1}{r_0} + x^2\sum\frac{x_k}{r_0^3} + xy\cancel{\sum\frac{y_k}{r_0^2}} + xz\cancel{\sum\frac{z_k}{r_0^2}} - \sum\frac{x_k}{r_0} - x\sum\frac{x_k^2}{r_0^3} - y\cancel{\sum\frac{x_k y_k}{r_0^2}} - z\cancel{\sum\frac{x_k z_k}{r_0^2}}\right.$$

$$-\frac{x^2}{2}\sum\left(-\frac{x_k}{r_0^3}+\frac{3x_k^3}{r_0^5}\right) - \frac{y^2}{2}\sum\left(-\frac{x_k}{r_0^3}+\frac{3x_k y_k^2}{r_0^5}\right) - \frac{z^2}{2}\sum\left(-\frac{x_k}{r_0^3}+\frac{3x_k z_k^2}{r_0^5}\right) *$$

$$\left. -3xy\cancel{\sum\frac{x_k^2 y_k}{r_0^5}} - 3yz\cancel{\sum\frac{x_k y_k z_k}{r_0^5}} - 3xz\cancel{\sum\frac{x_k^2 z_k}{r_0^5}}\right\} + 8Cx - 2C\sum x_k$$

$\frac{\varphi''}{2} \qquad -\varphi'' \qquad +\frac{\varphi''}{2}$

$$X = \cancel{\varphi' \frac{\partial r}{\partial x} + 2\varphi'(r-\ell)\frac{\partial r}{\partial x}}$$

$$B \frac{\partial r}{\partial x} + 2C\, r \frac{\partial r}{\partial x} + 3D\, r^2 \frac{\partial r}{\partial x}$$

$$\cancel{\Sigma X = \Sigma\{B}$$

$$\frac{\partial\left(\frac{1}{r}\right)}{\partial x} = \frac{x_k - x}{r^3}$$

$$\frac{\partial r}{\partial x} = -\frac{x_k - x}{r} \qquad \frac{\partial^2\left(\frac{1}{r}\right)}{\partial x^2} = -\frac{1}{r^3} + \frac{3}{r^5}(x_k - x)^2 \quad \Big\| \quad \frac{\partial^2}{\partial x \partial y} = \frac{3(x_k - x)(y_k - y)}{r^5}$$

$$\frac{1}{r} = \frac{1}{r_0} + x\left(\frac{-\frac{\partial r}{\partial x}}{r^2}\right)_0 \cdots + \frac{x^2}{2}\left(-\frac{1}{r^3} + \frac{3(x_k - x)^2}{r^5}\right)_0 + \cdots$$

$$+ xy\left(\frac{3(x_k - x)(y_k - y)}{r^5}\right)_0 + \cdots$$

$$= \frac{1}{r_0} + \frac{x x_k + y y_k + z z_k}{r_0^2} + \frac{x^2}{2}\left(-\frac{1}{r_0^3} + \frac{3x_k^2}{r_0^5}\right) + \frac{y^2}{2}\left(-\frac{1}{r_0^3} + \frac{3y_k^2}{r_0^5}\right) + \cdots$$

$$+3\left(\frac{xy\, x_k y_k}{r_0^5} + \cdots\right)$$

$$\Sigma B\left(\frac{x - x_k}{r}\right) = x \Sigma \frac{1}{r_0} - \frac{x(x^2+y^2+z^2)}{2} \Sigma \frac{1}{r_0^3} + \frac{3}{2}\left(x^3 \Sigma \frac{x_k^2}{r_0^5} + xy^2 \Sigma \frac{y_k^2}{r_0^5} + xz^2 \Sigma \ldots\right)$$

$$-\left\{x \Sigma \frac{x_k^2}{r_0^2} + y\, \cancel{\Sigma \frac{x_k y_k}{r_0^2}}\right.$$

$$\Sigma \frac{\partial r}{\partial x} = x(12 - 4\delta) - x \cdot 4(1+\delta) - \frac{x(x^2+y^2+z^2)}{2}\, 12(1-\delta) +$$

$$+ \frac{3}{2} 4\left[x^3\left(1 - \frac{\delta}{2}\right) + xy^2\left(1 - \frac{5\delta}{4}\right) + xz^2\left(1 - \frac{5\delta}{4}\right)\right]$$

$$= \cancel{x[8}\; 8x(1-\delta)$$

$\cancel{\Sigma r^2}$ $\quad 8(1+\delta)+4 = 12\left(1+\frac{2\delta}{3}\right)$

$\Sigma r^2 = 12\left[\left(1+\frac{2\delta}{3}\right) + (x^2+y^2+z^2)\right]$

$r^3 = r_0^3 + \cancel{3r_0^2 \frac{x x_k}{r_0}} + \frac{x^2}{2}\left[3r_0 + \frac{3x_k^2}{r_0}\right] + \frac{y^2}{2}[\quad] + \frac{z^2}{2}[\quad]$

$+ 3\,\frac{xy\; x_k y_k}{r} + \cdots$

$\Sigma \frac{x_k^2}{r_0} = 4(1+2\delta)\left(1-\frac{\delta}{2}\right) = 4\left(1+\frac{3\delta}{2}\right)$

$\Sigma \frac{y_k^2}{r_0} = 2\left(1-\frac{\delta}{2}\right)+2 = 4\left(1-\frac{\delta}{4}\right)$

$\Sigma \frac{z_k^2}{r_0} = 4\left(1-\frac{\delta}{4}\right)$

$\Sigma r^3 = 12(1+\delta) + \frac{3}{2}(x^2+y^2+z^2)\,12\left(1+\frac{\delta}{3}\right) + \frac{3}{2}\,4\left(1+\frac{3\delta}{2}\right)x^2 +$

$+ \frac{3}{2}\,4\left(1-\frac{\delta}{4}\right)(y^2+z^2)$

$= 12(1+\delta) + 18(x^2+y^2+z^2)\left(1+\frac{\delta}{3}\right) + 6x^2\left(1+\frac{3\delta}{2}\right) + 6(y^2+z^2)\left(1-\frac{\delta}{4}\right)$

$U = A + B\,\Sigma r + C\,\Sigma r^2 + D\,\Sigma r^3 \qquad = \alpha + \gamma\,[x^2+y^2+z^2]$

$\frac{\partial U}{\partial x} = B.8x + 2Cx + 18D.2x\left(1+\frac{\delta}{3}\right) + 12Dx\left(1+\frac{3\delta}{2}\right)$

$= x\left[8B + 2C + 48D + 30D\delta\right]$

$$\sum x_k^2 = 8\frac{(1+\delta)^2}{2} = 4(1+2\delta)$$

$$\sum \frac{1}{r_0} = 8 - 4\delta + 4 = 12 - 4\delta$$

$$\sum \frac{x_k^2}{r_0^3} = 4(1+2\delta)\left(1-\frac{3\delta}{2}\right) = 4\left(1+\frac{\delta}{2}\right)$$

$$\sum r_0 = 8\left(1+\frac{\delta}{2}\right) + 4 = 12\left(1+\frac{\delta}{3}\right)$$

$$\sum \frac{y_k^2}{r_0^3} = 2\left(1-\frac{3\delta}{2}\right) + 2 = 4\left(1-\frac{3\delta}{4}\right)$$

$$\sum \frac{z_k^2}{r_0^3} = 2\left(1-\frac{3\delta}{2}\right) + 2 = 4\left(1-\frac{3\delta}{4}\right)$$

$$\sum r_0^3 = 8\left(1+\frac{3\delta}{2}\right) + 4 = 12(1+\delta)$$

$$\sum \frac{x_k y_k}{r_0^3} = \cancel{(\ldots)} = 0$$

$$\sum \frac{1}{r_0^3} = 8\left(1-\frac{3\delta}{2}\right) + 4 = 12(1-\delta)$$

$$\sum r = \cancel{\ldots}\ 12\left(1+\frac{\delta}{3}\right) + \frac{x^2+y^2+z^2}{2}\, 12\left(1-\frac{\delta}{3}\right) -$$

$$- \frac{4}{2}\left\{x^2\left(1+\frac{\delta}{2}\right) + y^2\left(1-\frac{3\delta}{4}\right) + z^2\left(1-\frac{3\delta}{4}\right)\right\}$$

$$= 12\left(1+\frac{\delta}{3}\right) + x^2(6-2\delta-2-\delta) + y^2\left(6-2\delta-2+\frac{3\delta}{2}\right)$$

$$+ z^2\left(6-2\delta-2+\frac{3\delta}{2}\right)$$

$$\sum r = 12\left(1+\frac{\delta}{3}\right) + x^2(4-3\delta) + \cancel{\ldots} + (y^2+z^2)\left(4-\frac{\delta}{2}\right)$$ stimmt

$$\cancel{\ldots}\ \frac{\partial r^3}{\partial x} = 3r^2\frac{\partial r}{\partial x}$$

$$\frac{\partial^2}{\partial x^2} = 6r\left(\frac{\partial r}{\partial x}\right)^2 + 3r^2\frac{\partial^2 r}{\partial x^2} = 6r\left(\frac{x_k-x}{r}\right)^2 + 3r^2\left[\frac{1}{r} - \frac{(x_k-x)^2}{r^3}\right]$$

$$= 3r + 3\frac{(x_k-x)^2}{r}$$

$$\frac{\partial^2}{\partial x\partial y} = 6r\frac{\partial r}{\partial x}\frac{\partial r}{\partial y} + 3r^2\frac{\partial^2 r}{\partial x\partial y} = 6r\frac{(x_k-x)(y_k-y)}{r^2} - 3r^2\left[\frac{(x_k-x)(y_k-y)}{r^3}\right]$$

$$= \frac{3(x_k-x)(y_k-y)}{r}$$

$$r - l = r_0 - l + \frac{x^2 + y^2 + z^2}{r_0} + \frac{x^2}{2}\frac{\partial^2 r}{\partial x^2} + \frac{y^2}{2}\frac{\partial^2 r}{\partial y^2} + \frac{z^2}{2}\frac{\partial^2 r}{\partial z^2} + xy\frac{\partial^2 r}{\partial x \partial y} + \cdots$$

$$\frac{\partial r}{\partial x} = -\frac{x_k - x}{r}$$

$$\frac{\partial^2 r}{\partial x^2} = +\frac{1}{r} - \frac{(x_k - x)^2}{r^3} \quad \Big\| \quad \frac{\partial^2 r}{\partial x \partial y} = -\frac{(x_k - x)(y_k - y)}{r^3}$$

$$r - l = r_0 - l + \frac{3}{2}\frac{x^2 + y^2 + z^2}{r_0} - \frac{x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2 + 2z^2y^2}{2r_0^3}$$

$$r - l = r_0 - l \cancel{\frac{x(x-x_k) + y(y-y_k) + z(z-z_k)}{r}}$$

$$- x \cdot \left(\frac{x_k - x}{r}\right)_0 + \cdots + \frac{x^2}{2}\left\{\frac{1}{r} - \frac{(x_k - x)^2}{r^3}\right\}_0 + \cdots$$

$$r = r_0 - \frac{x x_k + y y_k + z z_k}{r_0} + \frac{x^2}{2}\left\{\frac{1}{r_0} - \frac{x_k^2}{r_0^3}\right\} + \frac{y^2}{2}\left\{\frac{1}{r_0} - \frac{y_k^2}{r_0^3}\right\} + \frac{z^2}{2}\left\{\frac{1}{r_0} - \frac{z_k^2}{r_0^3}\right\}$$

$$- \frac{xy\, x_k y_k}{r_0^3} \cdots$$

$$\cancel{r^2 = r_0^2 - 2(x x_k + y y_k + z z_k) + \frac{(x x_k + y y_k + z z_k)^2}{r_0^2} + x^2\left(1 - \frac{x_k^2}{r_0^2}\right) + \cdots}$$

$(x_k - x)^2 + \cdots$ $\quad -2(x_k - x)$ $\quad -\dfrac{2xy\, x_k y_k}{r_0^2}$

$$(r^2) = (r_0^2) - x \ldots 2(x_k \ldots \quad \cdots + x^2 + y^2 + z^2$$

$$(r^2) = (r_0^2) - 2(x x_k + y y_k + z z_k) + x^2 + y^2 + z^2$$

$$r_0 \left[1, 2, 3, 4, 9, 10, 11, 12\right] = \sqrt{\frac{(1+\delta)^2}{2} + \frac{1}{2}} = \sqrt{1 + \delta + \frac{\delta^2}{2}} = 1 + \frac{\delta}{2}$$

$$+ \frac{3}{2} - 2$$

$$\frac{1}{r_0} = 1 - \frac{\delta}{2}$$

$$\sum x_k = \sum y_k = \sum z_k = 0$$

$$r = 1 + \frac{\delta}{2} - \left(1 - \frac{\delta}{2}\right)\left(\frac{x(1+\delta)}{\sqrt{2}} - \frac{y_k}{\sqrt{2}}\right) + \frac{x^2 + y^2 + z^2}{2}\left(1 - \frac{\delta}{2}\right) - \left(1 - \frac{3\delta}{2}\right)\left[\frac{x^2}{4}(1+\delta)^2 + y^2\right]$$

$$x \cdot \frac{1 + 2\delta}{2 \cdot 2}\left(1 - \frac{3\delta}{2}\right)$$

1

$$U = \Phi_{(r)} = \Phi(l) + (r-l)\varphi + \frac{(r-l)^2}{2}\varphi' + \frac{(r-l)^3}{2.3}\varphi''$$

$R = -\frac{\partial U}{\partial r} = -\varphi$

dla sił odpychających φ ujemne

Rozciągnięcie w kierunku x:

$1 \begin{cases} x = \frac{1+\delta}{\sqrt{2}} l \\ y = -\frac{1}{\sqrt{2}} l \\ z = 0 \end{cases}$ $9 \begin{cases} x = -\frac{1+\delta}{\sqrt{2}} \\ y = \frac{1}{\sqrt{2}} \\ z = 0 \end{cases}$ $2 \begin{cases} x = \frac{1+\delta}{\sqrt{2}} \\ y = \frac{1}{\sqrt{2}} \\ z = 0 \end{cases}$ $10 \begin{cases} x = -\frac{1+\delta}{\sqrt{2}} \\ y = +\frac{1}{\sqrt{2}} \\ z = 0 \end{cases}$

$6 \begin{cases} x = 0 \\ y = \frac{1}{\sqrt{2}} \\ z = \frac{1}{\sqrt{2}} \end{cases}$ $8 \begin{cases} x = 0 \\ y = \frac{1}{\sqrt{2}} \\ z = -\frac{1}{\sqrt{2}} \end{cases}$ $5 \begin{cases} x = 0 \\ y = -\frac{1}{\sqrt{2}} \\ z = +\frac{1}{\sqrt{2}} \end{cases}$ $7 \begin{cases} x = 0 \\ y = -\frac{1}{\sqrt{2}} \\ z = -\frac{1}{\sqrt{2}} \end{cases}$

$3 \begin{cases} x = +\frac{1+\delta}{\sqrt{2}} \\ y = 0 \\ z = -\frac{1}{\sqrt{2}} \end{cases}$ $4 \begin{cases} x = +\frac{1+\delta}{\sqrt{2}} \\ y = 0 \\ z = \frac{1}{\sqrt{2}} \end{cases}$ $11 \begin{cases} x = -\frac{1+\delta}{\sqrt{2}} \\ y = 0 \\ z = -\frac{1}{\sqrt{2}} \end{cases}$ $12 \begin{cases} x = \frac{1+\delta}{\sqrt{2}} \\ y = 0 \\ z = \frac{1}{\sqrt{2}} \end{cases}$

$$U = \sum \left\{ \Phi(l) + \left[\sqrt{(x_k-x)^2+(y_k-y)^2+(z_k-z)^2} - l\right]\varphi + (\sqrt{} - l)^2 \frac{\varphi'}{2} + (\sqrt{} - l)^3 \frac{\varphi''}{2.3} \right\}$$

$$= \sum \left\{ \Phi - \varphi l + \frac{\varphi' l^2}{2} - \frac{\varphi'' l^3}{2.3} + \underbrace{\left(\varphi - l\varphi' + \frac{l^2\varphi''}{2}\right)}_{R} \sqrt{} + \left(\frac{\varphi'}{2} - \frac{\varphi''}{2} l\right) \sqrt{}^{\,2} + \frac{\varphi''}{2.3} \sqrt{}^{\,3} \right\}$$

$+ (\sqrt{} - l)^4 \frac{\varphi''}{4!}$

$$\left(\frac{1+\delta}{\sqrt{2}} - x\right)^2 + \left(-\frac{1}{\sqrt{2}} - y\right)^2 + z^2 = 1 + \delta \mp (1+\delta) x\sqrt{2} + y\sqrt{2} + x^2 y^2 z^2$$

$$(0-x)^2 + \left(-\frac{1}{\sqrt{2}} - y\right)^2 + \left(\frac{1}{\sqrt{2}} - z\right)^2 = 1 + y\sqrt{2} \mp z\sqrt{2} + x^2 y^2 z^2$$

$$16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = \left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta$$

$$f(\alpha) = \sqrt{\alpha} \qquad g'(\alpha) = \tfrac{1}{2}\sqrt{\alpha} - \sqrt{\alpha} = -\frac{\sqrt{\alpha}}{2}$$

$$g(\alpha) = -\frac{\sqrt{\alpha^3}}{3}$$

$$\frac{\alpha\sqrt{\beta}-\beta\sqrt{\alpha}}{i} - \frac{1}{3}\,\frac{\sqrt{\alpha^3}-\sqrt{\beta^3}}{i}$$

$$= r^{3/2}\left\{\sin\frac{\theta}{2} - \frac{1}{3}\sin\frac{3\theta}{2}\right\}$$

$$2\sin\frac{\theta}{2} - 2\sin\theta\,\sin\frac{\theta}{2}$$

$$= 2\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}$$

$$f'(\alpha) = \frac{1}{2\sqrt{\alpha}}$$

$$f''(\alpha) = -\frac{1}{4\sqrt{\alpha^3}}$$

$$\frac{1}{4}\left\{\frac{1}{\beta}-\frac{1}{\alpha}\right\} - \frac{1}{8}\cancel{\left\{\frac{1}{\sqrt{\beta\alpha}}\right\}} + \frac{1}{8}\left\{\sqrt{\frac{\beta}{\alpha^3}} - \sqrt{\frac{\alpha}{\beta^3}}\right\}$$

$$\int \log\beta\, d\beta = \beta(\log\beta - 1)$$

$$\sqrt{\alpha\beta^5} \qquad \frac{5}{2}\sqrt{\alpha\beta^3} \qquad \frac{15}{4}\sqrt{\alpha\beta}$$

$$\frac{15}{8}\sqrt{\frac{\beta}{\alpha}}$$

$$-\frac{15}{16}\sqrt{\frac{\beta}{\alpha^3}}$$

$$\psi = \frac{\alpha^2}{2}\left\{\beta\log\beta\right\} - \frac{\beta^2}{2}\left[\alpha\log\alpha\right] - \frac{8}{15}\left[\sqrt{\alpha\beta^5} - \sqrt{\beta\alpha^5}\right]$$

$$u = \frac{1}{i}\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) \qquad v = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta}$$

$$u = \frac{1}{i}\left\{\alpha\beta(\log\beta - \log\alpha) - \frac{1}{2}\left[\beta^2(\log\alpha+1) - \alpha^2(\log\beta+1)\right]\right\}$$

+	−	
4	6	−2
7	8	−1
7	5	+2
6	7	−1
4	8	−4
28	34	

0934   8921   7634   6232 | 4150

5051   ~~0628   9709   0086 | 3041~~

5883   ~~2293   9925   6456 | 0609~~

4472	4150	1139	9542
4449	3484	5093	4608

$$2\sqrt{\frac{64}{27}} = \frac{2.8}{5} = 3$$

$$\tau = \frac{M}{S} = \frac{M}{6\pi\mu R} = \frac{\frac{4}{3}\pi R^3 \rho}{6\pi\mu R} =$$

$$\tau = \frac{2}{9}\,\rho\frac{R^2}{\mu}$$

$$\frac{2}{9}\cdot\left(2{\cdot}5.10^{-6}\right)^2$$

$$\frac{2\pi\int_0^{\frac{\pi}{2}}\cos\theta\,\sin\theta\,d\theta}{2\pi} = \frac{1}{2}$$

Svedberg misst somit nur $\frac{1}{2}$ der wirkl. Beweg. grösse.

~~dazu kommt $\frac{1}{2}$ wegen Einführung der ganzen anstatt Viertelschwingungszeit~~

$$\cancel{\Delta = \frac{8}{3\sqrt{\pi}}\frac{1}{83.10^7}\sqrt{\;}}$$ $\sqrt{\pi}$ $c\sqrt{m} = 48000\sqrt{\frac{0{\cdot}0013}{4.10^{19}}}$

$$\Delta = \frac{8}{\rho\sqrt{\pi}}\frac{c\sqrt{m}}{\sqrt{2{\cdot}3.10^{-3}.\;2{\cdot}5.10^{-6}}}\sqrt{\frac{0{\cdot}032}{4}}\cdot\frac{1}{2}$$

$$= \frac{2}{9}\sqrt{\frac{0{\cdot}0013\cdot 0{\cdot}032}{10^{10}.\,2{\cdot}3.\,2{\cdot}5.10}}\cdot\frac{48000}{\sqrt{\pi}}$$

$$= \frac{2}{3}\sqrt{\frac{13.\cancel{32}}{\cancel{2{\cdot}3.\,2{\cdot}5.\,3{\cdot}4}}}\;10^{-5}.\,\cancel{48}\;16$$

$$5{\cdot}7 \qquad = \frac{32}{3}\sqrt{2{\cdot}2}\;.10^{-5} = 1{\cdot}6.\,10^{-4}$$

The Sowdby	4A	τ	η	$\frac{A}{\sqrt{\frac{\tau}{\eta}}}$	$\frac{\tau}{\tau}$	
Aceton	12·4	0·032	3·2	1·24	398	
Äthylacetat	7·8	0·028	4·6	1·00		28
Amylacetat	5·8	0·026	5·9	0·875		22
Wasser	4·2	0·013	10·2	1·18		32
n-Propylalk.	2·6	0·009	22·6	1·30		29

4472
6628
1·7844
89 21
0·8922

4150
7709
6441
3634
8220
9414

1139
0086
1053
6233
55265
0706

0·9542
2·3541
0·6001 −1
4150
0·3001
1150

$2·6. \sqrt{\frac{226}{9}}$ $\sqrt{25}$

$12·4 \sqrt{\frac{32}{3·2}}$

0·032

$$4A = \sqrt{\frac{\tau}{\eta}} \sqrt{\frac{RT}{N} \frac{1}{3\pi P}} = \sqrt{\frac{\tau}{\eta}} \sqrt{\frac{8·3 . 10^7 . 292}{4 . 10^{23}} \frac{100}{2·5 . 10^{-6}}}$$

$$= \sqrt{\frac{\tau}{\eta}} \; \frac{1}{2} \sqrt{\frac{10^9}{10^{17}}} = \frac{1}{2} \sqrt{\frac{\tau}{\eta}} \, 10^{-4} =$$

$$A = \frac{1}{8} \sqrt{\frac{\tau}{\eta}} . 10^{-4} =$$

$\frac{0·032}{0·0032}$

1038
580
6038

403

$\frac{0·032}{0·0023}$

5·051
− 3617
1·1434
5717
1038
6768

434

$\frac{0·009}{0·0226}$

9542
3541
1·6001
0·8000
1038
0·9038

801

$\frac{9}{13} \cdot \frac{10·2}{22·6} =$

$\frac{1}{1·44 . 2·2}$

$\frac{2·9}{4}$

$\sqrt{3·2} = 1·79$

$= 0·8$

Voigt Rapports I p. 320

$$-X_x = c_{11} x_x + c_{12} y_y + \dots \quad c_{16} x_y$$

$$-Y_z = c_{41} x_x + c_{42} y_y + \dots c_{44} y_z\ c_{45} \dots c_{46} x_y$$

Corps quasi isotrope :

~~Corps isotrope:~~

$$-X_x = c\, x_x + c_1 y_y + c_1 z_z$$

$$-Y_z = \frac{c_2\ y_z}{2}$$

$$\boxed{c_2 = c - c_1}$$

$$c = \frac{1}{5}(3A + 2B + 4C) \qquad c_1 = \frac{1}{5}(A + 4B - 2C) \qquad c_2 = \frac{1}{5}(2A - 2B + 6C)$$

où l'on a posé pour abréger

$$A = \frac{1}{3}(c_{11} + c_{22} + c_{33}) \qquad B = \frac{1}{3}(c_{23} + c_{31} + c_{12}) \qquad C = \frac{1}{3}(c_{44} + c_{55} + c_{66})$$

$$-X_x = c_{11} x_x = c_{11} \frac{\partial \xi}{\partial x}$$

$$-Y_y = c_{21} x_x = c_{21} \frac{\partial \xi}{\partial x}$$

$$-Y_z = c_{44} y_z = c_{44}\left(\frac{\partial \eta}{\partial z} + \frac{\partial \zeta}{\partial y}\right)$$

$$c = \frac{(1-\mu)}{(1+\mu)(1-2\mu)} E \; ; \; c_1 = \frac{\mu}{(1+\mu)(1-2\mu)} E \; ;$$

$$\frac{c}{c_1} = \frac{(1-\mu)}{\mu} = \frac{3A + 2B + 4C}{A + 4B - 2C}$$

$$\frac{c}{c_1} + 1 = \frac{1}{\mu}$$

$$\mu = \frac{1}{1 + \frac{c}{c_1}} = \frac{A + B - 2C}{4A + 6B + 2C}$$

$$-X_x = c\, x_x + c_1(y_y + z_z) = c\, x_x + (c - c_2)(y_y + z_z) = c\,(x_x + y_y + z_z) - c_2\,(y_y + z_z)$$

$$= c_2 x_x + (c - c_2)(x_x \dots)$$

$$-Y_z = \frac{c_2}{2} y_z$$

$r = 1 + \delta r$

$$\Big[1 + \delta r + \underbrace{\frac{3}{(1+\delta\omega)^2}}_{3(1-2\delta\omega)}\Big](3 + 3\delta\omega - 1) = 8(1+\delta\vartheta)$$

$$(4 + \delta r - 6\delta\omega)(2 + 3\delta\omega) = 8(1+\delta\vartheta)$$

$$8 + 2\delta r - \cancel{12\delta\omega} + \cancel{12\delta\omega} = 8(1+\delta\vartheta)$$

$\frac{T}{T_k} =$

$$\underline{\boxed{\delta r = 4\delta\vartheta}}$$

$$\vartheta = \frac{1}{8}\left(r + \frac{3}{\omega^2}\right)(3\omega - 1) = \frac{\left[(3\omega-1)r + \frac{9}{\omega} - \frac{3}{\omega^2}\right]}{8}$$

$$\vartheta = f(r=1, \omega=1) + \left(\frac{\partial f}{\partial r}\right)_{11} \cdot dr + \frac{\partial f}{\partial}$$

$$\left(\frac{\partial\vartheta}{\partial r}\right) = \left(\frac{3\omega-1}{8}\right) = \frac{1}{4} \qquad \left(\frac{\partial^2\vartheta}{\partial r^2}\right) = 0 \; ----$$

$$\frac{\partial^2\vartheta}{\partial r\,\partial\omega} = \frac{3}{8}$$

$$\Delta\theta = \frac{\Delta r}{4} + \frac{3}{8}\Delta r\,\Delta\omega + \frac{3}{8}\Delta\omega^3$$

$$\frac{\partial^3\vartheta}{\partial r\,\partial\omega^2} = 0 \; --$$

Izotermy $\quad \Delta r = \dfrac{-3\,\Delta\omega^3}{2 + 3\Delta\omega}$

$$\frac{\partial\vartheta}{\partial\omega} = \frac{1}{8}\left[3r - \frac{9}{\omega^2} + \frac{6}{\omega^3}\right]_0 = 0$$

$$\neq -\frac{3}{2}\Delta\omega^3$$

$$\frac{\partial^2\vartheta}{\partial\omega^2} = \frac{1}{8}\left[\frac{18}{\omega^3} - \frac{18}{\omega^4}\right] = \frac{9}{4}\left(\frac{1}{\omega^3} - \frac{1}{\omega^4}\right)_0 = 0$$

$$\cancel{\frac{\Delta r}{\Delta\theta} = 4\Delta\theta = -\frac{3}{2}\Delta\omega^3}$$

$$\Delta\omega^3 = \frac{8}{3}\Delta\theta$$

$$\frac{\partial^3\vartheta}{\partial\omega^3} = \frac{9}{4}\left(-\frac{3}{\omega^4} + \frac{4}{\omega^5}\right) = \frac{9}{4}$$

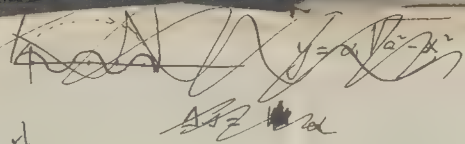
Ogólnie: $\quad \Delta r = \dfrac{8\Delta\theta - 3\Delta\omega^3}{2 + 3\Delta\omega} \neq 4\Delta\theta\,\cancel{\left(1 - \frac{3}{2}\Delta\omega\right) - \frac{3}{2}\Delta\omega^3}$

$$-\frac{3}{8}\Delta\omega^4$$

$$\int \Delta r\,\Delta\omega = 4\Delta\theta\,\Delta\omega - 3\Delta\theta\,\Delta\omega^2 = 0$$

$$4\Delta\theta - 3\Delta\theta\,\Delta\omega - \frac{3}{8}\Delta\omega^3 = 0$$

$\Delta s = \alpha^2 \beta \frac{\pi}{8}$



biorąc za długość fal: λ

$\beta\lambda = 2\pi$

$\beta = \frac{2\pi}{\lambda}$ $\Delta s = \frac{\alpha^2 \pi^2}{\lambda}$ nachylenie przeciętne: $\frac{\int \frac{dy}{dx} dx}{\int dx} = \frac{\alpha}{\lambda} = \varkappa$

$\Delta s = \varkappa^2 \lambda \pi^2$

$\Delta U = a \varkappa^2 \lambda \pi^2$

$$\overline{x^2} = \frac{\int x^2 e^{-a\lambda\pi^2 h x^2} dx}{\int - \, -} = \frac{1}{2a\lambda\pi^2 h}$$

$$h = \frac{N}{RT} = \frac{4 \cdot 10^{23}}{8 \cdot 10^7 \cdot 273} = \frac{10^{16}}{500} = 2 \cdot 10^{13}$$

$$\bar{x} = \frac{1}{\sqrt{2a\lambda\pi^2 h}}$$

$a = 80$

$\lambda = 10^{-4}$ (mikron)

$$\bar{x} = \frac{1}{\sqrt{160 \cdot 10 \cdot 10^{-4} \cdot 2 \cdot 10^{13}}} = \frac{1}{\sqrt{3 \cdot 10^{12}}} = \frac{1}{\sqrt{3}} \cdot 10^{-6}$$

$$h = \frac{1}{\pi f} \frac{a^2(\rho_1 - \rho_2)^2}{H} \qquad \frac{H}{\rho_1 - \rho_2} = c\left(1 - \frac{T}{T_k}\right)$$

$$= \frac{a^2}{\pi f} \cdot \frac{\rho_1 - \rho_2}{c\left(1 - \frac{T}{T_k}\right)}$$

h Bakker 20.7.54 für Äther 110°
= 8·5 $\mu\mu$

$$h_1 : h_2 = \frac{(\rho_1 - \rho_2)_0}{T_k - T} : \frac{\Delta\rho}{\Delta T}$$

$$h_1 \frac{\Delta\rho}{\Delta T} = h_2 \frac{\rho_1 - \rho_2}{T_k - T}$$

$$\Delta T = \frac{h_1}{h_2} \frac{\Delta\rho}{\rho_1 - \rho_2} \left(T_k - T\right)$$

197°
108°
89°

$\frac{h_1}{h_2} = \frac{1}{100}$

0·9°

197
273
470

$W\, d\alpha \sim e^{-hU}\, d\alpha$

$$\int e^{-hz^2} dz = \sqrt{\frac{\pi}{h}}$$

$$\int z^2 e^{-hz^2} dz = \frac{1}{2}\sqrt{\frac{\pi}{h^3}}$$

$$y = \alpha \sin \beta x$$

$$\frac{dy}{dx} = \alpha \beta \cos \beta x$$

$$s = \sqrt{1 + \alpha^2 \beta^2 \cos^2 \beta x}$$

$$\Delta s = \int_0^{\beta x = \frac{\pi}{2}} \left[\left(1 + \frac{\alpha^2\beta^2}{2}\cos^2\beta x\right) dx - dx\right] = \frac{1}{\beta}\int_0^{\varphi = \frac{\pi}{2}} \frac{\alpha^2\beta^2}{2}\cos^2\varphi \, d\varphi = \frac{\alpha^2\beta}{2}\,\frac{\pi}{4}$$

$$\frac{\Delta s}{\Delta x = \frac{\pi}{2\beta}} = \frac{\alpha^2\beta^2}{4}$$

$$\parallel \frac{\operatorname{tg}\varphi^2}{4}$$

$$\operatorname{tg}\varphi\big|_{x=0} = \alpha\beta$$

$$\Delta U = \frac{\alpha \operatorname{tg}^2\varphi}{4}$$

$$\operatorname{tg}\varphi = x$$

$$\overline{x^2} = \frac{\int x^2 e^{-hU} dx}{\int e^{-hU} dx} = \frac{\int x^2 e^{-\frac{h\alpha x^2}{4}} dx}{\int \cdots} = \frac{1}{2\,\frac{h\alpha}{4}} = \frac{2}{h\alpha}$$

$$n\frac{mc^2}{3} = RT$$
$$= \frac{p}{\rho}$$

$$\overline{\operatorname{tg}\varphi} = \sqrt{\frac{2}{h\alpha}}$$

$$h = \frac{2}{mc^2}$$
$$= \frac{2N}{3RT} = \frac{2N\rho}{3p}$$

$$= \sqrt{\frac{2}{3\alpha}} \quad 10^{-5}\cdot\sqrt{\frac{2}{3\alpha}}$$

$$h = \frac{2}{3}\,\frac{4\cdot 10^{19}\cdot 0.0013}{10^6} = 3\cdot 10^{10}$$

Etwa $\alpha = 20$

$$\sqrt{20} =$$

$$\overline{\operatorname{tg}\varphi} = 0.2\cdot 10^{-5} = (0.2\cdot(60)^3\cdot 10^{-5})'' = 0.4'' \quad (!)$$

7.

Wasilkowski Władysław: Prądy przemienne w zestawieniu z prądem o stałym kierunku. Sprawozdanie Dyrekcyi c.k. Gimnazyum VI. we Lwowie za rok szkolny 1903/4. Lwów 1904. Str. 32 i jedna tablica.

Czytam we wstępie: „.... wiem z doświadczenia, że brak czasu nie pozwala abituryentów dokładnie z tą rzeczą zaznajomić. Niech więc ta rozprawka będzie dla nich uzupełnieniem tego, co słyszeli w gimnazyum z nauki o indukcyi elektrycznej. A może i który z nauczycieli będzie mi wdzięczny za tę pracę."

Ładne zadanie i dobre zamiary! Niestety jednak, przeczytałem rozprawkę i zamiast wdzięczności uczułem żal do autora. Trudno bowiem o rozprawkę, napisaną gorzej, niż książeczka p. Wasilkowskiego. Nie chciałbym być satyrycznym, ale pomimo to mogę powiedzieć, iż jest to zbiór usterek i błędów.

I tak zaraz na początku (str. 3.) można ze zdumieniem czytać, że maszyna dynamo wytwarza prądy, zmieniające co chwila swój kierunek. I cóż powie na to biedny abituryent, którego nauczono w gimnazyum, że dynamo dostarcza prądu o kierunku stałym? — Lub dalej (str. 6.) dowiadujemy się, że „sama obecność prądu galwanicznego w jednym łączniku wzbudza w drugim drucie, nie mającym żadnej łączności z pierwszym, prąd elektryczny." Tymczasem dziesięć wierszy przedtem jest mowa o tem, że obecność prądu, byle on stale płynął, wcale nie może wywołać prądu indukowanego. — Jednorodnem polem magnetycznem nazywa prof. Wasilkowski to pole (str. 14.), „w którem linie siłowe (?) są do siebie równoległe," nie mówiąc nic o tem, jak rzęsiście w rozmaitych miejscach uważanej przestrzeni linie owe są rozsiane. — Rozróżnienie dzielności od pracy jest tak nieściste, że dzielność wyraża autor (str. 23.) kilogramometrami lub siłą (sic!) koni.

I takie fatalne omyłki spotykamy co krok. Nie chcę mnożyć liczby podobnych przykładów; może już te, które podałem, wystarczą do tego, aby czytelnik wyrobił

II B

Budowa

Atomu

z rysunkami